STRIBECK AND TRACTION CURVES FOR ELLIPTICAL CONTACTS: ISOTHERMAL FRICTION MODEL

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Abstract: This paper deals with the prediction of the Stribeck and traction curves, by proposing a mixed lubrication model for highly loaded elliptical contacts. The model represents an extension on the mixed lubrication model of Gelinck and Schipper and comprehends both the asperity component or the so called boundary lubrication component (BL) and the elastohydrodynamic component (EHL). The asperity component is calculated from a fully deterministic contact perspective, where an equivalent rough surface is in contact with a smooth and rigid surface. In EHL regime, the film thickness is calculated according to Nijenbanning et al. and the separation for the asperity component is derived from Johnson, with a small adaptation, which was possible due to the deterministic contact model approach. In this way, the separation is calculated from the volume conservation theory of Johnson and even if this can be negative, the film thickness remains above zero permitting the calculation of highly loaded contacts. For the traction curve calculation, an elastic-plastic approximation for BL micro-contacts as proposed by Gelinck and Schipper for line contacts is used.

Keywords: Mixed lubrication, Friction model, Stribeck curve, Traction curve.

1 INTRODUCTION

The tool which controls friction between interacting surfaces is lubrication, which has the effect of decreasing friction, making the lubricated mechanism more reliable. In another perspective, lubrication is present as a negative effect, in places where friction should be high, like traction systems, i.e. wheel-rail contacts.

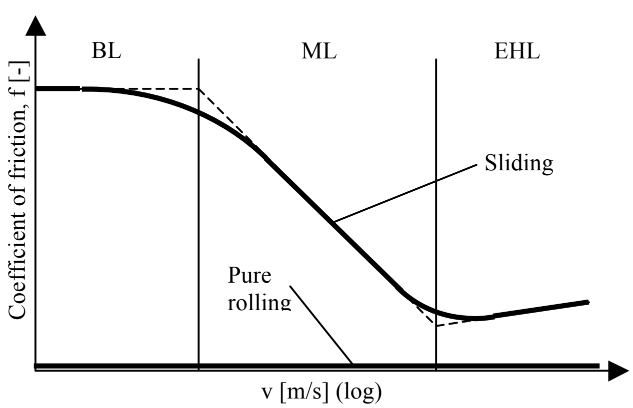

Fig. 1 Generalized Stribeck curve.

For rolling to sliding contacts, three lubrication regimes are distinguished: the boundary lubrication (BL) regime, where the velocity between the contacting bodies is low and the load is carried solely by interacting asperities of the opposing surfaces, the elasto-hydrodynamic lubrication (EHL) regime, at high velocity, where the load is carried by the pressure generated in the lubricant (in this case friction is controlled by shearing the lubricant) and the transition regime between these two lubrication modes is represented by the mixed lubrication (ML) regime where the load is carried by the asperities as well as the pressure generated in the lubricant. All three regimes are schematically visualized in Fig. 1 in which friction is depicted as a function of the sum velocity of the components in contact. In this figure friction for the sliding and for the rolling contact situation is shown. For pure rolling, due to the absence of sliding only rolling friction is present which is one of more orders lower compared to sliding friction.

The variation of the coefficient of friction with velocity, over the above presented lubrication regimes, represents the so called Stribeck curve. For the prediction of the Stribeck curve for line contacts, Gelinck and Schipper [1] used the elastic contact model of Greenwood and Williamson [2] for the asperity contact component and Moes function fit [3] for the central film thickness in EHL component. The model was further extended to highly loaded line contacts by Faraon [4], where the asperity component was solved deterministic, using Zhao's elasto-plastic model [5] and the separation was calculated by using the volume conservation model of Johnson [6].

The present paper extends the previous mentioned model for elliptical contacts including the slip component over the entire lubrication regimes. Next, by using an elastic plastic shear model for boundary layer, traction curves are predicted in which the coefficient of friction is represented as a function of roll/slide (slip) ratio.

2 MIXED LUBRICATION MODEL

2.1 Elastohydrodynamic component

For the hydrodynamic component, the first parameter to calculate is the film thickness. In order to simplify the calculation and to have a small number of variables, dimensionless numbers were introduced.

The first set of dimensionless numbers is defined after Dowson and Higginson [7]:

$$G = \alpha E' ;\ \bar{h} = \frac{h_c}{R_x} ;\ U_\Sigma = \frac{\eta_0 v^+}{E' R_x} ;\ W = \frac{F_N}{E' R_x^{\ 2}} \tag{1}$$

where the number G is referred to the lubricant, $\bar{h}$ to the film thickness, U_Σ to the velocity and W to the load respectively.

The second set consists of three numbers and are defined after Moes [3]:

$$H = \bar{h} U_\Sigma^{-\frac{1}{2}} ;\ M = W U_\Sigma^{-\frac{3}{4}} ;\ L = G U_\Sigma^{\frac{1}{4}} \tag{2}$$

where H is the film thickness number, M is the load number and L is the lubricant number.

Using these numbers, the system of three equations which describes an EHL elliptic contact (Reynolds equation, film shape and load balance) was solved numerically by Nijenbanning et al. [8] resulting in a film shape and pressure distribution.

It must be mentioned that in the Reynolds equation, the viscosity of the lubricant was considered to vary with pressure according to Roelands equation [9], which reads:

$$\eta(p) = \eta_\infty \exp\left\{ \left[\left(1 + \frac{p}{p_r} \right)^z - 1 \right] \ln\left(\frac{\eta_0}{\eta_\infty} \right) \right\} \tag{3}$$

After computing the results, an equation for the central film thickness was derived by curve fitting the numerical results using the dimensionless numbers:

$$H_C = \left\{ \left[H_{RI}^{\frac{3}{2}} + \left(H_{EI}^{-4} + H_{00}^{-4} \right)^{-\frac{3}{8}} \right]^{\frac{2s}{3}} + \left(H_{RP}^{-8} + H_{EP}^{-8} \right)^{-\frac{s}{8}} \right\}^{\frac{1}{s}} \tag{4}$$

$$s = 1.5 \left[1 + \exp\left(-1.2 \frac{H_{EI}}{H_{RI}} \right) \right] \tag{5}$$

where, H_{00} is a constant, H_{RI} is the rigid-isoviscous asymptote, H_{EI} the elastic-isoviscous asymptote, H_{RP} the rigid-piezoviscous asymptote and H_{EP} the elastic-piezoviscous asymptote.

$$H_{00} = 1.8 \cdot \vartheta^{-1} \tag{6}$$

$$H_{RI} = 145 \cdot \left(1 + 0.796 \cdot \vartheta^{\frac{14}{15}} \right)^{-\frac{15}{7}} \vartheta^{-1} M^{-2} \tag{7}$$

$$H_{EI} = 3.18 \cdot \left(1 + 0.006 \cdot \ln \vartheta + 0.63 \cdot \vartheta^{\frac{4}{7}} \right)^{-\frac{14}{25}} \cdot \vartheta^{-\frac{1}{15}} M^{-\frac{2}{15}} \tag{8}$$

$$H_{RP} = 1.29 \cdot \left(1 + 0.691 \cdot \vartheta \right)^{-\frac{2}{3}} L^{\frac{2}{3}} \tag{9}$$

$$H_{EP} = 1.48 \cdot \left(1 + 0.006 \cdot \ln \vartheta + 0.63 \cdot \vartheta^{\frac{4}{7}} \right)^{-\frac{7}{20}} \cdot \vartheta^{-\frac{1}{24}} M^{-\frac{1}{12}} L^{\frac{3}{4}} \tag{10}$$

where $\vartheta = R_x / R_y$ is the ratio of the reduced radius of curvatures in and perpendicular to the rolling/sliding direction.

2.2 Asperity component

The surface is measured with an interferometer and then it is exported in a way it can be used in a microcontact model. Using the 9 point summit definition [10], every asperity is computed as ellipsoids determined by height z_i and two radii of curvature in the main directions. The contact between asperities and the counter surface is assumed to be elastic and the contact area and normal load is determined according to the Hertz theory.

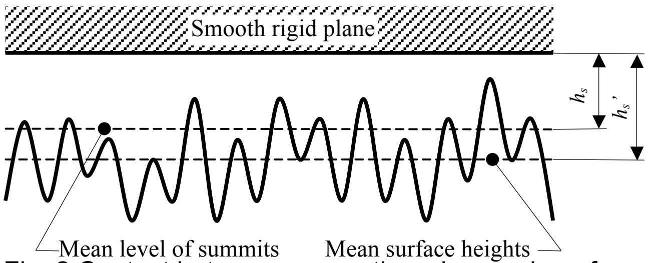

Fig. 2 Contact between a smooth and a rough surface.

The asperity contact area A_C and the load carried by asperities F_C are given by summation of the individual contributions of each asperity:

$$F_C = \sum_{i=1}^{N} F_i\left(w_i\right) \tag{11}$$

$$A_C = \sum_{i=1}^{N} A_i\left(w_i\right) \tag{12}$$

where $w_i = z_i - h_s$ is the asperity indentation or deformation depth and N is the number of asperities in contact.

2.3 Separation

In order to calculate the separation in the BL regime the film thickness according to Johnson et al. [6] is used. Here the film thickness is defined by the average fluid volume between two rough surfaces divided by nominal contact area. The statistical formulation of the above mentioned film thickness proposed by Johnson is:

$$h_c = \int_{-\infty}^{h_s} \left(h_s - z\right) f\left(z\right) dz \tag{13}$$

where h_c is the film thickness, h_s is the separation and $f\left(z\right)$ is the Gaussian distribution of the asperities. With this definition, the volume of the deformed asperities is ignored, by limiting the surface points coordinate to a maximum equal with the separation. A different approach is proposed here, where the volume of the deformed asperities also preserves. The deterministic formulation is:

$$h_c = \frac{1}{n} \sum_{j=1}^{n} \left(h_s - z_j\right) \tag{14}$$

where n is the number of heights and z_j is the height relative to center line average (CLA). The equivalent statistical definition is:

$$h_c = \int_{-\infty}^{\infty} \left(h_s - z\right) f\left(z\right) dz \tag{15}$$

Here, the volume described by the deformed material is taken into account assuming that the material is incompressible and its volume preserves as well. This assumption is made for the separation calculation, the asperity contact model is calculated according to the Hertz theory.
Using this definition, even if the separation can become negative, the film thickness always remains positive.

2.4 The model

Mixed lubrication is the transition regime between BL and EHL. Here, the coefficient of friction takes values between the coefficient of friction of the other regimes and the load is carried by the BL and the EHL force component:

$$F_N = F_C + F_H \tag{16}$$

Where F_C is the load carried by asperities and F_H is the load carried by lubricant.

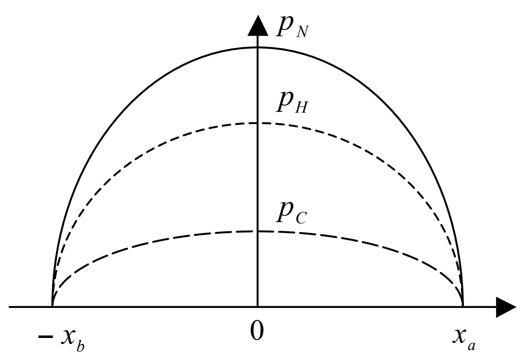

Fig. 3 Pressure distribution in an ML contact.

In terms of pressure, Eq. 16 is written as [6]:

$$p_N = p_C + p_H \tag{17}$$

Based on this equation, two coefficients γ_1 and γ_2 are introduced [6]:

$$\gamma_1 = \frac{F_N}{F_H} = \frac{p_N}{p_H} \text{ and } \gamma_2 = \frac{F_N}{F_C} = \frac{p_N}{p_C} \tag{18}$$

The above defined coefficients are mutually dependent through Eq. 19.

$$\frac{1}{\gamma_1} + \frac{1}{\gamma_2} = 1 \tag{19}$$

Using these two coefficients and combining a deterministic multi asperity contact model with the EHL theory, the entire Stribeck curve can be computed.

The hydrodynamic component is calculated using the γ_1 coefficient. Making the substitutions [6], $F_N \to F_N/\gamma_1$ and $E' \to E'/\gamma_1$ in Eq. 4-5 the central film thickness for an elliptical contact becomes:

$$H_C = \gamma_1^{\frac{1}{2}} \left\{ \left[\gamma_1^{\frac{9}{4}} H_{RI}^{\frac{3}{2}} + \left(\gamma_1^{-\frac{2}{5}} H_{EI}^{-4} + H_{00}^{-4} \right)^{-\frac{3}{8}} \right]^{\frac{2s}{3}} + \left[\gamma_1^{4} \left(H_{RP}^{-8} + H_{EP}^{-8} \right) \right]^{-\frac{s}{8}} \right\}^{\frac{1}{s}} \qquad (20)$$

$$s = 1.5 \left[1 + \exp\left(-1.2 \frac{H_{EI}}{H_{RI}} \gamma_1^{-\frac{7}{5}} \right) \right] \qquad (21)$$

The constant H_{00} and the four asymptotes H_{RI}, H_{EI}, H_{RP} and H_{EP} remains the same as presented in Eq. 6-10. The dimensionless film thickness H_C is given by Eq. 2.
The pressure carried by the asperities is calculated from the load carried by the asperities Eq. 11 and the real contact area Eq. 12:

$$p_C = \frac{F_C}{A_C} \qquad (22)$$

Knowing the mixed lubrication regime, means knowing the load carried by asperities F_C, the load carried by the lubricant F_H and the film thickness h_C. In order to find these values, a system of three equations has to be solved:

- Load balance Eq. 16.
- Film thickness Eq. 14 using Eq. 20 in which the definition of Johnson [6] is used.
- Finally, the pressure calculated from the deterministic microcontact model p_C given by Eq. 22 must be equal with the central mean pressure of the elliptical contact.

Solving this system of three equations with three unknowns, the coefficient of friction in ML can be computed as presented further. The solution scheme of the solver is presented in Appendix A.

2.5 Friction force

The velocity between two moving surfaces can be regarded as a superposition of a pure rolling and a pure sliding motion. In Fig. 4 are plotted the individual velocities of the moving surfaces as well as their sum $v^+ = v_1 + v_2$ and their difference or the sliding velocity $v^{dif} = |v_1 - v_2|$. In the same figure, lines for constant sliding velocity, constant sum velocity and constant slip are drawn. The slip or slide-to-roll ratio is defined as:

$$S = \frac{v^{dif}}{v^+} 200\% = \frac{|v_1 - v_2|}{v_1 + v_2} 200\% \qquad (23)$$

In Fig. 4 (b), if the coefficient of friction is plotted perpendicular to the velocity field, each constant slip line in the velocity field represents a Stribeck curve. A traction curve (most frequently used in EHL) represents the coefficient of friction evolution by varying the slip, while keeping the sum velocity constant.

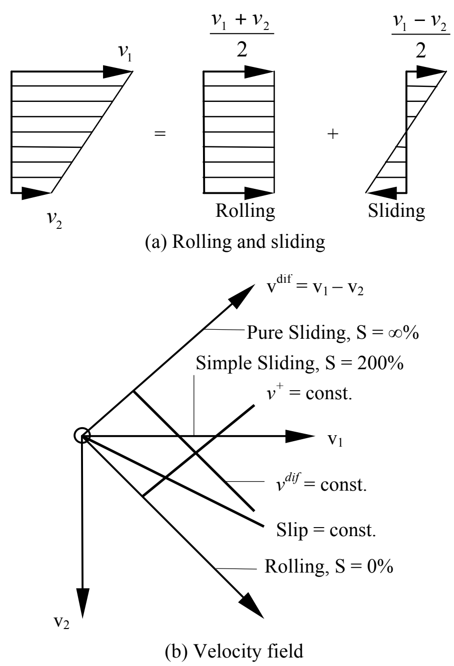

Fig. 4 Velocity field for two moving surfaces [11].

The friction force in EHL lubricated contacts is caused by shearing the lubricant. In BL, the friction force is caused by shearing the boundary layer existing between the contacting asperities. The level of friction is often expressed by the so called shear rate $\dot{\gamma}$ defined as:

$$\dot{\gamma} = \frac{v^{dif}}{h_c} \qquad (24)$$

where h_C is the separation between the opposing surfaces.

2.5.1 Friction in EHL

In the EHL regime, friction is caused by shearing the lubricant in the contact. The shear stress τ_H in the lubricant is defined as a function of the shear rate $\dot{\gamma}$ and using the Eyring model, this is:

$$\tau_H = \tau_0 \operatorname{arcsinh}\left(\frac{\eta\dot{\gamma}}{\tau_0}\right) \quad (25)$$

Integrating the shear stress over the contact area, the friction force is obtained:

$$F_{f,EHL} = \iint_{A_H} \tau_H(\dot{\gamma})\, dA_H \quad (26)$$

where $A_H = A_{nom} - A_C$ is the hydrodynamic contact area. The friction force generated by the EHL component is expressed by:

$$F_{f,EHL} = \tau_0 A_H \operatorname{arcsinh}\left(\frac{\eta S v^+}{2h_c}\right) \quad (27)$$

2.5.2 Friction in BL

The friction in BL regime is caused by shearing the boundary layer between the contacting asperities. Integrating the shear stress τ_C over the asperity contact area, the friction force generated by a pair of asperities in contact is obtained. So, the friction force in the boundary lubrication regime is equal with the sum of all contacting asperities:

$$F_{f,BL} = \sum_{i=1}^{N} \iint_{A_{Ci}} \tau_{Ci}\, dA_{Ci} \quad (28)$$

Considering the friction force in the asperity contact to be of Coulomb type i.e. $\tau_C = f_{Ci} \cdot p_{Ci}$, with a constant value of the coefficient of friction f_C, the friction force in the BL regime for the simple sliding situation is written:

$$F_{f,BL} = f_C \sum_{i=1}^{N} \iint_{A_{Ci}} p_{Ci}\, dA_{Ci} = f_C F_C \quad (29)$$

where the Coulomb coefficient of friction f_C is experimentally determined.

Eq. 29 gives the value of the friction force in BL for simple sliding contacts. However, in rolling to sliding contacts, taking into account the slip for the BL regime, there is no theoretical model. The slip is incorporated in the shear rate, but since in the presented model the friction in the asperity component is considered to be of Coulomb type, the shear stress of the boundary layer does not change with the shear rate. Therefore, an arctangent function fit as used by Gelinck [12] for the traction curve in BL for line contacts is also used here.

$$f = f_C \frac{2}{\pi} \arctan\left(\frac{\pi}{2}\frac{S}{S_{ep}}\right) \quad (30)$$

A generalized traction curve calculated with Eq. 30 is shown in Fig. 5.

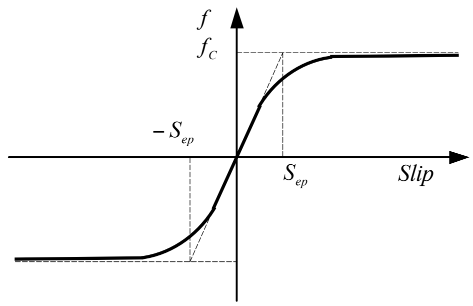

Fig. 5 Schematic traction curve in BL.

When operating in the BL regime and increasing the slip, while the sum velocity is kept constant, the coefficient of friction will increase from zero when $S = 0\%$ (pure rolling) to the Coulomb value f_C when $S = 200\%$ (simple sliding). A parameter S_{ep} is introduced, which is a measure for the transition from elastic to plastic behavior of the boundary layer. This parameter is determined experimentally.

Combining Eq. 29 and Eq. 30 will result in an expression for the friction force in the BL regime:

$$F_{f,BL} = f_C F_C \frac{2}{\pi} \arctan\left(\frac{\pi}{2}\frac{S}{S_{ep}}\right) \quad (31)$$

2.5.3 Friction in ML

The friction force in the ML regime is given by summing the friction force generated by the asperities in contact and the friction generated by shearing the lubricant:

$$F_{f,ML} = F_{f,BL} + F_{f,EHL} \quad (32)$$

Substituting Eq. 27 and Eq. 31 in Eq. 32 the formula for the coefficient of friction in ML is determined:

$$f = \frac{f_C F_C \frac{2}{\pi} \arctan\left(\frac{\pi}{2}\frac{S}{S_{ep}}\right) + \tau_0 A_H \operatorname{arcsinh}\left(\frac{\eta S v^+}{2\tau_0 h_c}\right)}{F_N} \quad (33)$$

After solving the ML regime (load carried by asperities, load carried by lubricant and the separation), the coefficient of friction can be computed using Eq. 33 over the entire range of lubrication regimes.

If the slip S is kept constant and the sum velocity v^+ is varied, the result of the presented ML friction model, finalized with Eq. 33, is a Stribeck curve. If the sum velocity is kept constant and the slip is varied, the output of the model is the traction curve.

3 RESULTS AND DISCUSSIONS

3.1 Stribeck curve

The results of the friction calculations are presented in Fig. 6. For comparison, the calculation for a line contact, for the same contact pressure is given. The line contact is validated with experiments [4]. Different results can be obtained from the model, as real contact area, number of asperities in contact, load carried by the asperities, load carried by the lubricant, separation between the surfaces and most important the variation of the coefficient of friction as a function of velocity.
The input parameters used in the calculation are presented in Table 1.

Table 1 Input parameters.

Property	Value	Unit	Description
n	$1.1\cdot10^{11}$	$1/m^{-2}$	Density of asperities
β	$8.3\cdot10^{-6}$	m	Average radius of asperities
σ_s	$7.6\cdot10^{-8}$	m	Standard deviation of asperities
E’	231	GPa	Combined elasticity modulus
η_0	0.02	Pa·s	Viscosity
τ_0	2.5	MPa	Eyring shear stress
α	$2\cdot10^{-8}$	Pa^{-1}	Viscosity – pressure coefficient
z	0.679	-	Viscosity – pressure index
R_x	$10\cdot10^{-3}$	m	Radius in x (rolling) direction
R_y	$40\cdot10^{-3}$	m	Radius in y direction
F_N	100	N	Normal load
f_c	0.13	-	Coefficient of friction in BL

3.1.1 Model check by comparing with equivalent line contact

In Fig. 6 (Stribeck curves) are plotted the coefficient of friction (left y axis) and dimensionless separation, h_s/σ, σ standard deviation roughness, (right y axis) as a function of velocity within the specified range. In general, the coefficient of friction decreases in the ML regime as the film thickness increases with velocity.
Nijenbanning [8] proposed an equivalent line contact for comparing the film thickness given by Eq. 4 with the film thickness calculated for line contacts [3]. A similar model is proposed here but slightly different. For a given elliptical contact, the equivalent line contact is the contact having the same half width in the rolling direction and the same mean contact pressure at the same normal load. So for given R_x, R_y and F_N, the equivalent line contact will be dimensioned as follows:

$$B_c = \frac{F_N}{2\cdot a_x\cdot p_m} \tag{34}$$

$$R_c = \frac{\pi\cdot E'\cdot a_x}{16\cdot p_m} \tag{35}$$

Using the above dimensions for the line contact with the same normal load as used in the elliptical situation, the mean contact pressures and the half widths in rolling direction in both situations will be equal. The dimensions of the equivalent cylinder for the input geometry from Table 1, according to Eq. 34 and 35 are: $B_c = 0.6$ mm and $R_c = 10.8$ mm.

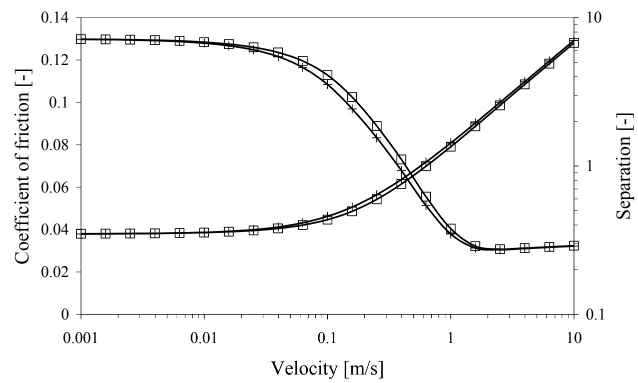

Fig. 6 Comparison between the elliptical contact (□) and the equivalent line contact (+).

The results are very close. There is a small difference for the coefficient of friction at the transition between BL and ML regime.

3.1.2 Influence of load

As can be noticed in Fig. 7, the transition between the different lubrication regimes is hardly influenced by the normal load, the ML regime however extends over a wider velocity range and the coefficient of friction is lower with increasing load in the low velocity region.

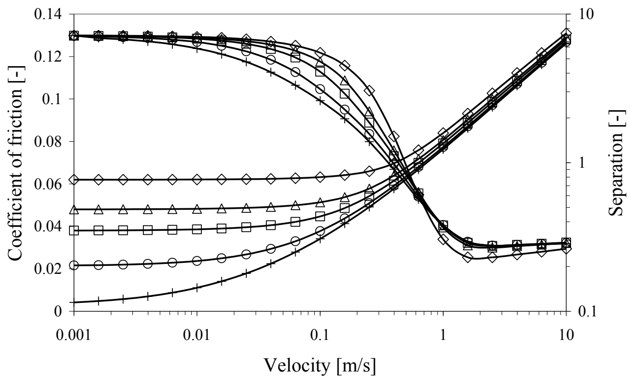

Fig. 7 Stribeck curve and separation for different normal loads: ◊ 10 N (281 MPa), Δ 50 N (481 MPa), □ 100 N (606 MPa), ○ 200 N (764 MPa) and + 300 N (874 MPa).

3.1.3 Influence of roughness

In order to study the influence of the surface roughness on the coefficient of friction and separation curve, several surfaces with Gaussian heights distribution were randomly generated and associated with the same macro geometry and external factors as presented in Table 1.

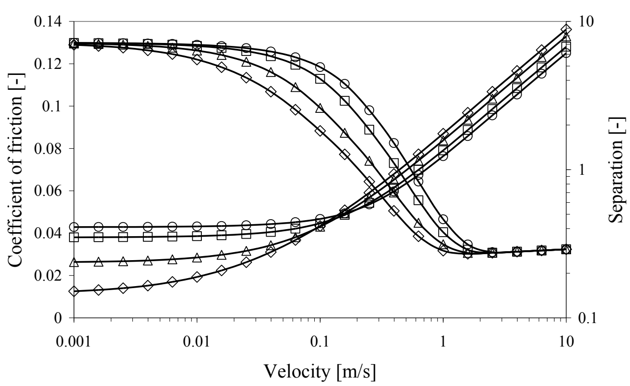

Fig. 8 Effect of the statistical roughness parameters σ_s (standard deviation of asperity heights) on Stribeck curve and separation; standard deviation of roughness of: ◊ 0.06, Δ 0.07, □ 0.08 and ○ 0.09 μm respectively.

In Fig. 8 it can be noticed that the Stribeck curve shifts to the right with increasing roughness, which makes sense because when the standard deviation of the roughness increases, then the separation between the two surfaces has to be larger to have lift off (full separation between the surfaces).
Because the presented model is deterministic, the roughness influence discussion is possible with different generated surfaces using as input the standard deviation. In which case, for each generated surface, the average radius of the summits is decreasing with decreasing the standard deviation of the asperity heights distribution. In the four situations presented above the average radius of the summits decreased from 11 microns for the smooth surface to 7 microns for the rough one.

3.1.4 Influence of viscosity

The same surface and inputs were chosen as presented in Table 1 to study the effect of viscosity on the Stribeck curve for elliptical contacts.

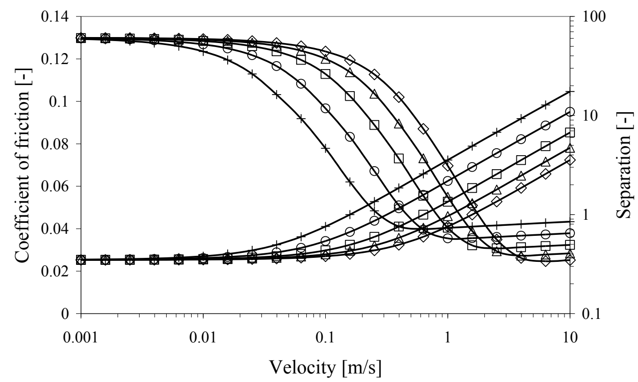

Fig. 9 Influence of viscosity η_0 on the Stribeck curve and separation. Viscosity of ◊ 0.008, Δ 0.012, □ 0.02, ○ 0.04 and + 0.08 Pas respectively.

As expected, the film thickness at the same velocity is higher when a more viscous lubricant is considered. Therefore the ML regime shifts to the left while the friction in the EHL regime increases due to increasing of viscosity.

3.2 Traction curve

In Fig. 10 the coefficient of friction is plotted as a function of slip for different values of sum velocity.

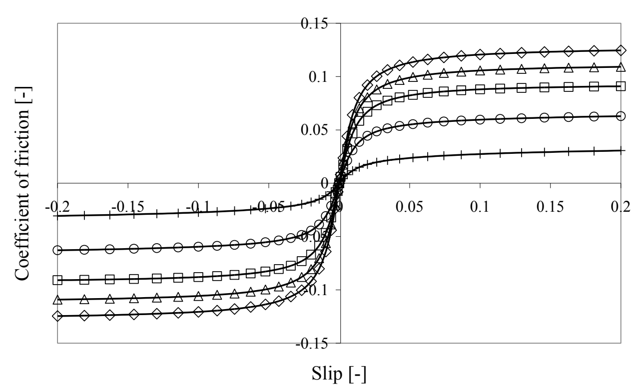

Fig. 10 Traction curves at different velocities. Sum velocity of ◊ 0.01, Δ 0.1, □ 0.2, ○ 0.4 and + 1 m/s respectively.

The input parameters for this calculation are presented in Table 1.
As can be seen the coefficient of friction is increasing from 0 (pure rolling) until it reaches the corresponding value for simple sliding as the calculated Stribeck curve Fig. 6 – Elliptical contact. At low velocity (0.01 m/s) the coefficient of friction is increasing from nearly zero to 0.13 after the slip is exceeding approximately 5%.

In another paper the effect of heat generation in the contact on friction, Stribeck curve and traction curve, is discussed [13].

4 CONCLUSIONS

A mixed lubrication friction model is developed for lubricated elliptical contacts. A good agreement is found between the presented elliptical model and the proposed equivalent line contact. Different parameters were varied in order to study their influence on the Stribeck curve and separation. In the present case, the variations noticed were similar to the existing friction model for line contacts. The most significant influences in the Stribeck curve are in the ML regime which shifts to the left (the coefficient of friction decreases) with increasing the normal load and the viscosity of the lubricant and with decreasing surface roughness. Not less important is the EHL regime which shifts up with increasing both normal load and viscosity of the lubricant. Of course, the roughness in EHL regime has no influence on the friction level. Finally, since the friction in BL regime is of Columbian type, none of the above parameters influences the friction level in that regime.

NOMENCLATURE

a_x = half width of the contact area in rolling direction, [m]
A_C = real contact area, [m^2]
A_H = hydrodynamic area, [m^2]
A_{nom} = nominal contact area (Hertz), [m^2]
B_c = width of the equivalent cylinder, [m]
E' = combined elasticity modulus, [Pa]
f = coefficient of friction, [-]
f_C = coefficient of friction in BL, [-]
F_C = load carried by asperities, [N]
F_H = load carried by lubricant, [N]
$F_{f,\,BL}$ = friction force in the contacting asperities, [N]
$F_{f,\,EHL}$ = friction force in the lubricant, [N]
$F_{f,\,ML}$ = friction force in ML regime, [N]
F_N = total normal load, [N]
G = dimensionless lubricant number, [-]
$\bar{h}$ = dimensionless film thickness, [-]
h_s = separation, [m]
h_c = central film thickness, [m]
H = dimensionless film thickness, [-]
L = lubricant number, [-]
M = load number, [-]
N = number of contacting asperities, [-]
h_c = central film thickness, [m]
p = pressure, [Pa]
p_0 = ambient pressure, [Pa]
p_m = mean contact pressure, [Pa]
p_C = pressure carried by asperities, [Pa]
p_H = pressure carried by lubricant, [Pa]
p_N = pressure exerted by the normal load, [Pa]
p_r = constant (p_r=1.962×10^8), [Pa]
R_c = radius of the equivalent cylinder, [m]
R_x = combined radius in x (rolling) direction, [m]
R_y = combined radius in y direction, [m]
S_0 = viscosity - temperature index, [-]
S = slip, [-]
S_{ep} = slip at the transition from elastic to plastic behaviour of the boundary layer, [-]
T_0 = reference temperature, [°C]
T = temperature, [°C]
$tg(\alpha_v)$ = slope of the Eyring shear stress function of velocity, [-]
$tg(\alpha_p)$ = slope of the Eyring shear stress function of pressure, [-]
U_Σ = dimensionless velocity number, [-]
v^+ = sum velocity, [m/s]
v^{dif} = sliding velocity, [m/s]
W = dimensionless load number, [-]
z = viscosity - pressure index, [-]

Greek symbols

α = viscosity - pressure coefficient, [-]
$\dot{\gamma}$ = Shear rate $\dot{\gamma}=v^{dif}/h_c$, [1/s]
$\gamma_{1,2}$ = Johnson's factors, [-]
ϑ = ratio of the reduced radius of curvatures, [-]
η = viscosity, [Pa·s]
η_0 = viscosity at ambient pressure, [Pa·s]
$\eta_{¥}$ = constant ($\eta_{¥}$=6.315×10^{-5}), [Pa·s]
τ_{00} = reference shear stress, [Pa]
τ_0 = Eyring shear stress, [Pa]
τ_H = shear stress of the lubricant, [Pa]

Abbreviations

BL = Boundary Lubrication
ML = Mixed Lubrication
EHL = Elasto Hydrodynamic Lubrication
RI = Rigid/Isoviscous asymptote
EI = Elastic/Isoviscous asymptote
RP = Rigid/Piesoviscous asymptote
EP = Elastic/Piesoviscous asymptote

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Appendix A

Solving the ML regime means in fact, determining the load carried by the asperities. This is done by following the solution scheme in Fig. 11.

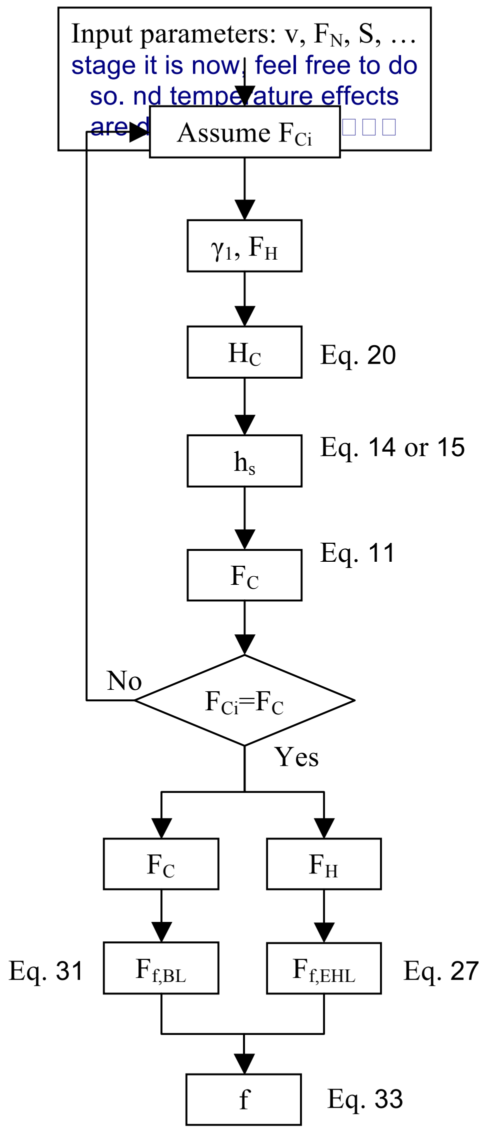

Fig. 11 Solution scheme for the calculation of the friction coefficient in ML regime.

REFERENCES

[1] Gelinck, E.R.M. and Schipper, D.J. (2000), “Calculation of Stribeck Curves for line contacts”, Tribology International 33, 175-181.

[2] Greenwood, J.A. and Williamson, J.B.P. (1966), “Contact of nominally flat surfaces”, Phil Trans R Soc London, Series A, 295, 300-319.

[3] Moes, H. (2000), “Lubrication and Beyond”, lecture notes, University of Twente, Enschede, NL, www.tr.ctw.utwente.nl/research/publications.

[4] Faraon, I.C. (2005), “Mixed lubricated line contacts”, Ph.D. Thesis, University of Twente, Enschede, The Netherlands.

[5] Zhao, Y., Maietta, D.M. and Chang, L. (2000), “An asperity microcontact model incorporating the transition from elastic deformation to fully plastic flow”, ASME Journal of Tribology 122, 86-93.

[6] Johnson, K.L., Greenwood, J.A. and Poon, S.Y. (1972), “A simple theory of asperity contact in elastohydrodynamic lubrication”, Wear 19, 91-108.

[7] Dowson, D. and Higginson, G.R. (1966), “Elasto-hydrodynamic lubrication – The fundamentals of roller and gear lubrication”, Pergamon Press, Oxford.

[8] Nijenbaning, G., Venner, C.H. and Moes, H. (1994), “Film thickness in elastohydrodynamically lubricated elliptic contacts”, Wear 176, 217–229.

[9] Roelands, C.J.A. (1966), “Correlation aspects of the viscosity – temperature – pressure relationship of lubricated oils”, Ph.D. thesis, Technische Hogeschool Delft, The Netherlands.

[10] Rooij, M. (1998), “Tribological aspects of unlubricated deepdrawing processes”, Ph.D. thesis, University of Twente, Enschede, The Netherlands.

[11] Schipper, D.J. (1988), “Transitions in the Lubrication of Concentrated Contacts”, Ph.D. thesis, University of Twente, Enschede, The Netherlands.

[12] Gelinck, E.R.M. (1999), “Mixed lubrication of line contacts”, PhD thesis, University of Twente Enschede, The Netherlands.

[13] Popovici, R.I. and Schipper, D.J., “Prediction of Stribeck and traction Curves for highly loaded Elliptical Contacts”, accepted for the World Tribology Conference WTC 2013, Torino, Italy.