

A COMPARISON OF FATIGUE LIFETIME PREDICTION MODELS APPLIED TO VARIABLE AMPLITUDE LOADING

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Abstract: The loads imposed on, amongst others, offshore structures can vary considerably with time. Lifetime prediction methodologies need to consider possible acceleration and retardation of the crack growth rate due to load sequences. Models based on a linear accumulation of damage will have a limited accuracy and are not considered as a sufficiently valuable asset in lifetime prediction of structures subjected to variable amplitude loading. This necessitates more complex nonlinear damage evolution models that can be applied in a so-called cycle-by-cycle analysis. In this paper, a comparison is made between three cumulative damage models (Miner, modified Miner and weighted average) and two yield zone models (Wheeler and Willenborg). Experimental data of fatigue crack growth in offshore steel specimens subjected to sequential loading is used as basis of the comparison. The modified Miner model is the most promising of the cumulative damage models but the determination of the parameter α requires laboratory tests. Evaluation of the effects of variation in the model input parameters on estimated lifetime reveals a large influence for the Miner and weighted average approaches.

Keywords: fatigue; variable amplitude; Miner; Wheeler; Willenborg

1 INTRODUCTION

Fatigue is known to be the main cause of failure of cyclically loaded structures (aerospace, automotive, offshore...) [1]. Offshore structures, for example, are subjected to combinations of loads caused by wind, currents and waves [2]. Due to these natural phenomena, stresses in offshore structural components fluctuate in time as illustrated in Figure 1.

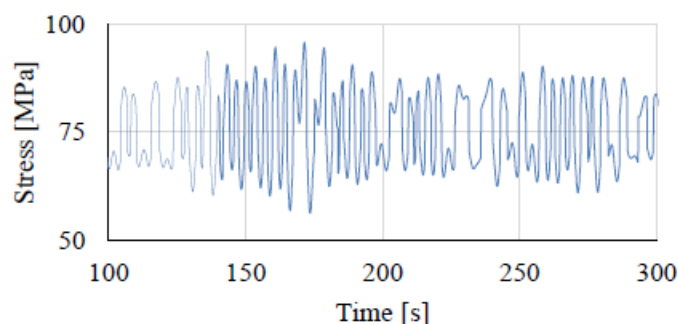


Figure 1: Hot spot stress as function of time in an offshore structural component [3]

In variable amplitude fatigue, load and interaction effects come into play [4]. Load effects represent the linear component of the damage evolution in time: the higher the applied stress, the higher the damage and vice versa. Interaction effects cover the non-linear component of the damage evolution; acceleration or retardation effects arise due to a sudden change in the stress level. A closer look at the stress spectrum in fig. 1 shows that besides changes in stress amplitude also large variations in mean stress are present. Consequently, offshore structures are expected to be more prone to interaction effects [5].

Notwithstanding the efforts in this research domain, not all influences are fully understood so far. The following conclusions resulted from previous work [4, 6, 7, 8]. The typical result of an overload is a retardation of the crack growth. An acceleration of the crack growth can be observed following an underload. A combination of an overload and an underload will in most cases result in retardation. The underload strongly influences the amount of retardation in this case. For block loading, the results depend on the block amplitude, block length and the stress ratio (R) applied.

Fatigue lifetime prediction models developed for constant amplitude (CA) cyclic loading fail to accurately represent the fatigue crack growth process in variable amplitude (VA) conditions [9]. Therefore, specific VA

models have been developed. These models are expected to take the load and interaction effects into account but come at the expense of higher complexity.

This paper will first present three cumulative damage models, namely Miner, modified Miner and weighted average model and afterwards two yield zone models, namely Wheeler and Willenborg. Using [10] as reference and the results of the variable amplitude tests reported in [11], a comparison of the accuracy in lifetime prediction of the five models will be discussed.

2 VARIABLE AMPLITUDE LIFETIME PREDICTION: CUMULATIVE DAMAGE MODELS

2.1 Palmgren-Miner

The Palmgren-Miner rule predicts the lifetime of a specimen that is subjected to cyclic loading by calculating the damage parameter D [12] that yields values between 0 (no damage) and 1 (complete failure). If partial damages D_i ($i = 1, \dots, N$) are imposed to the specimen by N different sources, the total damage is calculated as

$$\sum_{i=1}^N D_i = D \quad (1)$$

The ratio $\frac{D_i}{D}$ is the fractional damage imposed by the i -th source. Failure occurs when the damage parameter reaches a predefined critical value less than or equal to one. In the conventional endurance approach, this damage concept is applied by considering the number of cycles n_i applied for the corresponding stress level σ_i .

The Palmgren-Miner rule predicts failure when

$$\sum_{i=1}^N \frac{n_i}{N_i} = 1 \quad (2)$$

with N_i the number of cycles to failure when a CA fatigue stress level of σ_i is applied.

In the fracture mechanics approach, the concept of using a ratio of number of cycles as fractional damage cannot be used directly. To allow comparing different models, a definition of fractional damage is needed that can be consistently used. In this paper, the critical value of D is defined to correspond to the maximum crack growth realized in a fatigue test. This allows to use equation 2, with the value for N_i determined as the number of cycles needed to obtain the critical crack growth at a certain constant stress intensity factor range ΔK_i .

For VA loading conditions an effective stress intensity factor range ΔK_{eff} is determined which allows to calculate an effective number of cycles N_{eff} that can be compared to the number of cycles needed to obtain the critical crack growth in an experiment. The following equation is used to implement the Palmgren-Miner rule.

$$\Delta K_{eff} = \sum_{i=1}^N \frac{n_i}{N_i} \cdot \Delta K_i \quad (3)$$

This approach is known as the Linear Cumulative Damage (LCD) model. The number of cycles n_i at each ΔK_i level is determined using a cycle counting technique like the rainflow method [7]. The LCD model does not consider variable amplitude effects as no specific parameters are used to take the influence of loading sequences into account. Consequently, this approach will yield the best results in case of VA fatigue for symmetrical spectra in which retardation and acceleration effects balance each other.

2.2 Modified Miner

The here presented modified Miner approach is an adaptation of Miner's rule or the LCD model to take VA effects into account. Their physical background is the same; a comparison is made between the applied number of cycles at a certain stress level and the total number of cycles to failure at that stress level. A nonlinear evolution of damage is assumed by introducing a model parameter α that depends on the load sequence [2]. This results in the so-called Non-Linear Cumulative Damage (NLCD) model. The formula to calculate the effective stress intensity factor range is

$$\Delta K_{eff} = \sum_{i=1}^N \left(\frac{n_i}{N_i} \right)^\alpha \cdot \Delta K_i \quad (4)$$

This modification of Miner's rule allows predicting the number of cycles a specimen will last under VA load more accurately. The best results are still obtained for random spectra where the different VA effects balance each other [9].

2.3 Weighted average

In literature, the modified Miner rule or the NLCD model and the weighted average model are often considered to be the same [7, 9]. This paper keeps both models separated. The weighted average model is based on a mathematical average of the stress intensity factor ranges applied during a load sequence. The effective stress intensity factor range is calculated as

$$\Delta K_{eff} = \left[\sum_{i=1}^N \frac{n_i}{N_T} \cdot (\Delta K_i)^\beta \right]^{1/\beta} \quad (5)$$

with N_T the total number of cycles applied and β a model parameter to account for VA effects [7]. A value for β equal to 2 allowed Barsom to accurately predict fatigue crack growth rate in steels [6, 7, 13]. The effective stress intensity factor range calculated for β equal to 2 is defined as the root mean square effective stress intensity factor range [6]. Statistical fits to a wide range of test data resulted in a β value approximately equal to 3 [7, 14]. The corresponding effective stress intensity factor range is known as the root mean cube effective stress intensity factor range.

The results for this model are comparable to the modified Miner model. Excellent results are obtained for highly irregular load spectra where the VA effects balance each other [6].

3 VARIABLE AMPLITUDE LIFETIME PREDICTION: YIELD ZONE MODELS

The following two models are defined as yield zone models [15, 16]. They consider the plastic zone in front of the crack tip to describe the interaction effect.

3.1 Wheeler

The Wheeler model is based on the damage accumulation concept but uses a simple retardation parameter C_p as presented in equation 6

$$a = a_0 + \sum_{i=1}^n C_p f(\Delta K_i, r_i, \dots) \quad (6)$$

where C_p varies between 0 and 1 depending on the location of the crack tip in the previously formed plastic zone (r_p on Figure 2) and the plastic zone size of the current load cycle r_i .

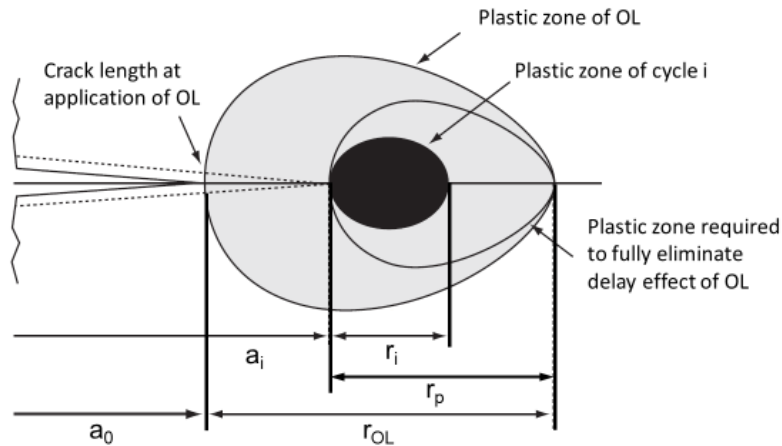


Figure 2: Plastic zone dimensions at crack tip [16]

Plastic zone size r_p is calculated using the following equation

$$r_p = a_{OL} - a_i \quad (7)$$

with

$$a_{OL} = a_0 + r_{OL} \quad (8)$$

This allows to calculate the value of C_p using the following equations

$$C_p = \left[\frac{r_i}{r_p} \right]^w \quad \text{when } r_i < r_p \quad (9)$$

$$\text{and } C_p = 1 \quad \text{when } r_i \geq r_p$$

where r_i is the current plastic zone size, r_{OL} is the overload plastic zone size, a_{OL} is the crack length at overloading and exponent w is a function of the stress level, the crack shape and the load spectrum. Wheeler used the Irwin plane stress relation for the plastic zone computation. Contrary to the assumption of Wheeler, a different value of w should be used for each loading spectrum [15]. A single value of w would lead to imprecise estimates of the crack propagation rate [16]. The Wheeler model has two known limitations, i.e. crack arrest cannot be predicted and delayed retardation will not be recognized [15].

This model successfully predicts crack growth retardation due to single overloads in a constant amplitude spectrum. Underloads however are not taken into account. Also the combination of overloads and underloads cannot be solved using this model [9, 17, 18].

3.2 Willenborg

Willenborg described crack growth retardation as a function of the stress intensity factor needed to overcome the plastic zone created by an overload. The model assumes a reduction of the effective stress at the crack tip due to residual compressive stresses produced by the overload. This effect is represented by a reduction of the stress intensity factor by a constant K_{red} , calculated using

$$K_{red} = K_{req} - K_{max} \quad (10)$$

with K_{req} the stress intensity factor required to produce a plastic zone corresponding with the zone of dimension r_p on Figure 2 and K_{max} the maximum stress intensity factor corresponding to the CA loading for a crack length a_i [16]. An effective R-ratio can be calculated as follows:

$$R_{eff} = \frac{K_{min,eff}}{K_{max,eff}} = \frac{K_{min} - K_{red}}{K_{max} - K_{red}} \quad (11)$$

and an effective stress intensity factor range ΔK_{eff} is calculated as:

$$\Delta K_{eff} = K_{max,eff} - K_{min,eff} \quad (12)$$

These parameters are used to calculate the crack growth extension for each cycle. When the crack grows outside the plastic zone that is induced by the overload, the residual stress and the retardation effect are ignored.

As the Willenborg model does not use an empirical factor m , this model is preferred over the Wheeler model. A drawback of the Willenborg model is that crack arrest is predicted at a constant value of the overload ratio $R_{OL} = K_{max,OL}/K_{max} = 2$ [15, 19]. This ratio is however known to depend on the material and the load [15].

Only retardation is taken into account in the Willenborg model. Consequently, symmetric spectra will not lead to accurate predictions. The Willenborg model obtains a high level of accuracy in situations with in general retardation effects. The applicability of the model for predicting the retardation effect depends on the level of overloading [16].

Gallagher [20] tried to improve the residual stress and crack arrest assumptions and formulated the generalized Willenborg model. This model is restricted to retardation effects. Brussat [21] tried to solve this problem and suggested a modification to the generalized Willenborg model. He tried to take the reduction of the retardation effect due to an underload into account by introducing an additional model parameter ϕ . Chang and Engle [21] suggested a modification to take acceleration due to an underload into account [15, 19].

Implementation of the Wheeler and the Willenborg model requires iterative calculation and a cycle-by-cycle determination of the stress intensity factor range. Consequently, applying these models is much more complex than the application of Miner's rule and its related variations.

4 EVALUATION OF MODELS

4.1 Comparison of model lifetime predictions and experimental data

Fatigue crack growth tests have been performed using eccentrically loaded single edge notched specimens (ESE(T)) made of steel grade NV F460 [11]. The Paris law parameters are summarized in Table 1.

Table 1: Paris law constants

	C	m
F460	4.00E-12	3.064

The tests performed are ΔK controlled tests consisting of 15 subsequent blocks. Each block is characterized by its own stress intensity factor range ΔK_i and a target crack growth Δa_i . The test is continued at a constant stress intensity factor range as long as the target crack growth is not reached. This procedure is graphically represented in Figure 3.

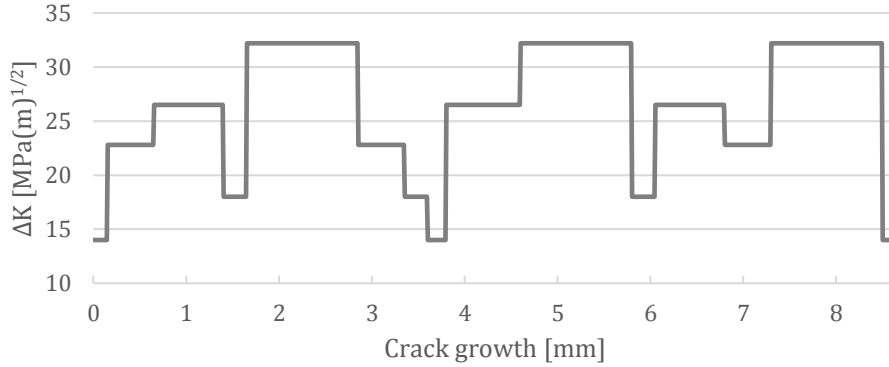


Figure 3: Loading sequence: applied stress intensity factor range versus crack growth

The final crack growth is set as the state at which a critical value of damage is reached. The final crack extension Δa_f is used to calculate the number of cycles N_i defined in section 2.

$$N_i = \frac{\Delta a_f}{\left(\frac{da}{dn}\right)_i} \quad (13)$$

The total number of cycles applied N_T , needed in the weighted average approach, is determined as:

$$N_T = \sum_{i=1}^N n_i \quad (14)$$

An effective number of cycles N_{eff} that corresponds to the effective stress intensity factor range ΔK_{eff} can be calculated. Substituting this ΔK_{eff} in the Paris law, the effective crack growth rate $(da/dn)_{eff}$ can be determined.

$$\left(\frac{da}{dn}\right)_{eff} = C \cdot (\Delta K)_{eff}^m \quad (15)$$

Dividing the critical crack growth by this effective crack growth rate gives the effective number of cycles.

$$N_{eff} = \frac{\Delta a_f}{\left(\frac{da}{dn}\right)_{eff}} \quad (16)$$

Figure 5 presents the predicted number of cycles per method. The result obtained with Miner's rule is much smaller than the measured number of cycles. The here presented modified Miner approach allows to find an equal number of cycles as measured during the test by optimizing the parameter α . The optimal value of α for this material and this load sequence is equal to 1.198. Application of the weighted average model with the recommended values for β resulted in a lifetime prediction that is much more accurate than the Miner approach, but still far from the actual value. The results could be further improved by decreasing β , and the best results were obtained by using a value for β that is positive but as low as possible.

One important remark should be taken into account for the modified Miner model. Finding an accurate α while doing a post-mortem analysis is quite straightforward. A material related α -value is however impossible to determine without prior knowledge of the load sequences. The accuracy of the modified Miner model as function of α is plotted in Figure 4. Accuracy is defined as the ratio of the predicted number of cycles obtained from the model and the number of cycles that resulted from the tests, presented as a percentage. Small variations in the value of parameter α can clearly lead to major differences in lifetime prediction.

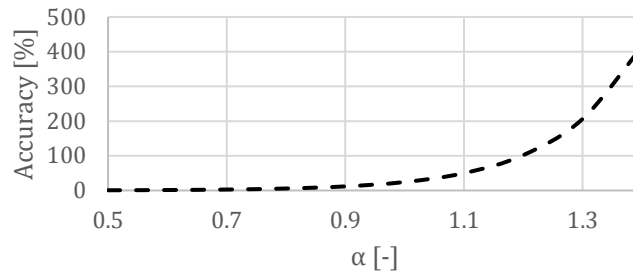


Figure 4: Modified Miner accuracy as function of α

A next comparison is made using the Wheeler and Willenborg models and is also presented in Figure 5. As the experimental results used in this paper only showed retardation and crack arrest, the Wheeler and Willenborg model should allow an accurate prediction of the effective number of cycles [22].

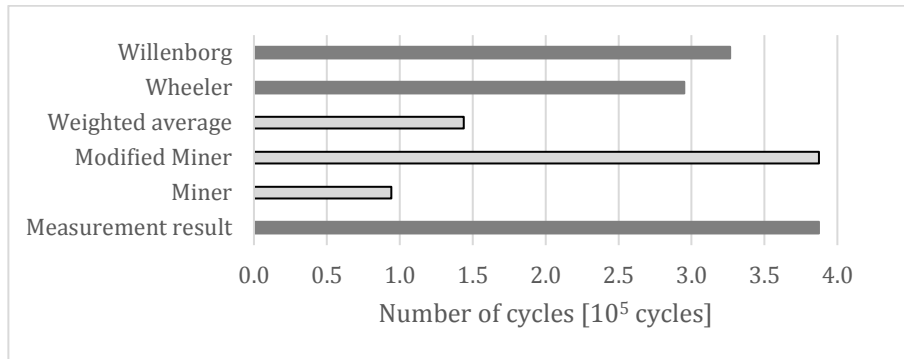


Figure 5: Predicted number of cycles for material NV F460

As can be seen, the results for the yield zone models are more accurate than the cumulative damage models, except for the modified Miner result since this last model was tuned to fit the experimental results.

4.2 Influence of variation in model input parameters on lifetime prediction

The fracture mechanics models are based on the Paris law curve characterized by parameters C and m . These parameters are obtained by fitting curves to data clouds. Inherently some uncertainty will be present in the input of models studied in the previous section. The influence of variations in these input parameters is studied in this section.

Figure 6 shows the reference Paris law curve together with its variations that have been used. The original values for parameters C and m are respectively $4 \cdot 10^{-12}$ and 3.064. Variations of $\pm 10\%$ of both parameters are examined. The accuracy results obtained for the Paris law variants are presented in Table 2.

Table 2: Accuracy of results obtained for Paris law variants [%]

	Miner	Modified Miner	Weighted average
Original	24.32	100.00	47.28
C +10%	22.11	90.91	42.99
C -10%	27.02	111.11	52.54
m +10%	8.91	42.20	20.18
m -10%	66.38	236.99	112.59

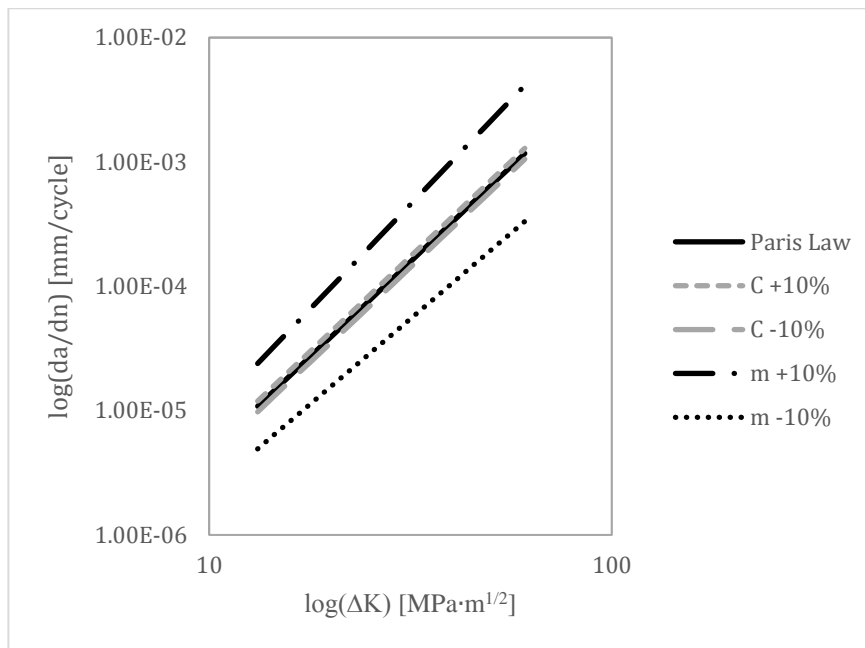


Figure 6: Studied variations of Paris law curve

The results for the modified Miner approach are mentioned above for completeness. They are obtained by keeping α constant. This results in values that can be larger than 100% due to the used definition of accuracy. All values can however be brought back to 100% by changing the used α -value between 1.063 and 1.309. Consequently, the trends shown for this approach are of minor importance.

The accuracy of the Miner and weighted average approach increases when the values of the parameters are reduced and decreases when they become larger. For both the influence of variation in parameter C is limited but the influence of variation in parameter m is more significant.

5 CONCLUSIONS

This paper compared cumulative damage models and yield zone models in fatigue lifetime prediction applications. The cumulative damage models can be used to have a conservative insight in the lifetime of a specific structure under variable amplitude loading. The degree of conservatism is reduced when using the weighted average or the modified Miner model.

Variation of the Paris law parameters C and m influences the accuracy of the lifetime prediction. If the values of the parameters increase, the accuracy of the prediction decreases and vice versa. The influence of variation of m is more significant than the influence of variation of C .

The results of the yield zone models, Wheeler and Willenborg, are more accurate than those of the cumulative damage models and not influenced by an unknown parameter.

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