EVALUATION OF PIPE BENDING REFERENCE STRESS EQUATIONS

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Abstract The use of a Failure Assessment Diagram (FAD) is widespread in the assessment of weld defects. To determine whether a defect is acceptable or not, this requires the calculation of a load ratio and a fracture ratio for the defect under consideration. Nowadays, many formulae are available to calculate these two quantities and no clear guidance is given on which equation(s) should (not) be used. A partial clarification of this problem is achieved by comparing different reference stress equations. This article is concerned with such comparison, for the specific case of welded pipes subjected to a bending load. A large set of historical experimental data has been investigated in which defected pipes were subjected to an increasing bending force until failure occurred. Two kinds of reference stress equations are considered, full pipes subjected to a bending load and flat plates subjected to a uniform tension load. From the equations under consideration, the flat plate solution of Goodall & Webster and the empirical full pipe equations of Willoughby and Wilkowski & Eiber describe the pipe failure in the most accurate way.

Keywords Failure Assessment Diagram, Reference stress, Pipe bending, Defects

1 INTRODUCTION

Girth welds of pipelines unavoidably contain defects. Whether these defects are acceptable or not is, in codes and standards, often determined by using a Failure Assessment Diagram (FAD) [1, 2]. This paper focuses on the assessment prescribed in the general European FITNET procedure [3].
Using a FAD, two values have to be calculated in order to determine the severity of a defect. The first value is the fracture ratio (K_r), which is determined as the ratio of a stress intensity factor to the material's fracture toughness. The stress intensity factor can be calculated by different available formulae [3, 4]. In this paper, only the Newman and Raju equation (see Appendix B) will be used. The second value is the load ratio (L_r), determined as the ratio of a reference stress to the material's yield strength. The reference stress can be calculated according to many formulae, each of them applicable to a specified load condition and geometry. If the calculated failure assessment point (L_r, K_r) is located underneath the failure assessment curve, the defect is considered to be acceptable (see Figure 1).

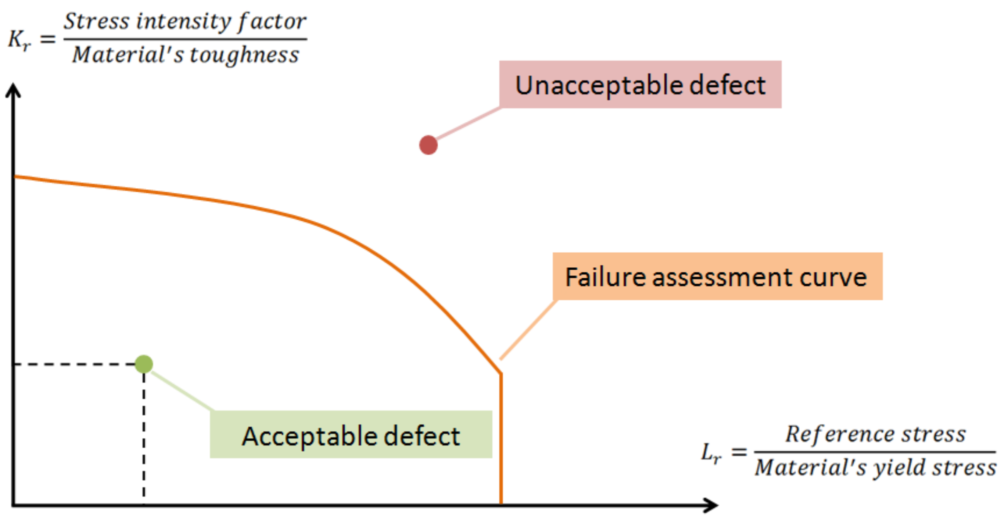

Figure 1. The Failure Assessment Diagram (FAD) can be used to determine the acceptability of a defect

In the open literature, many equations are available to calculate the reference stress, which makes it complicated to decide which equation should be used. Therefore a comparison of reference stress equations was made for pipes subjected to a bending moment, based on a large amount of full scale test data [5, 6]. The equations under investigation were derived either for full pipes subjected to a bending load or for flat plates subjected to a uniform tension load. This paper briefly describes the used reference stress

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equations and then compares them based on the historical experimental data. Final conclusions provide advice on which equation should be used for the considered case.

2 REFERENCE STRESS EQUATIONS

Following a literature review, several reference stress equations have been selected. The following paragraphs give a brief description of these equations together with their limitations. The defect dimensions are defined according to Figure 2.

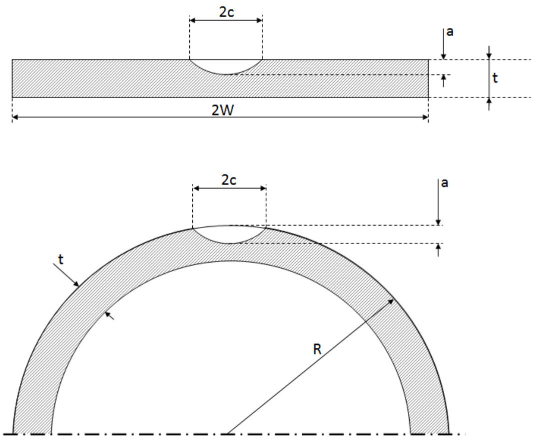

Figure 2. Pipe, plate and defect dimensions used in limit load equations

The pipes are loaded as illustrated in Figure 3. From the applied bending moment, the bending stress can be calculated in two ways, elastically ($\sigma_{b,e}$) and plastically ($\sigma_{b,p}$):

$$\sigma_{b,e} = \frac{M}{\pi \cdot R^2 \cdot t} \tag{1}$$

$$\sigma_{b,p} = \frac{M}{4 \cdot R^2 \cdot t} \tag{2}$$

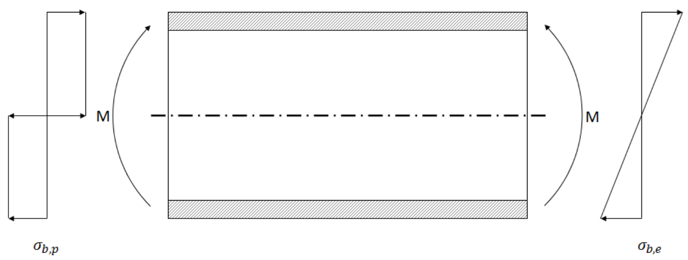

Figure 3. Applied load

The reference stress equations can be divided into two subgroups. On the one hand reference stress can be calculated using equations for entire pipes subjected to bending. On the other hand reference stress equations obtained from flat plates subjected to a uniform tensile stress are discussed. This second category makes sense since the pipe section containing the defect is in the tension loaded region of the pipe. Besides for large D/t-ratios this section is subjected to a (nearly) constant stress and the curvature of the plate is limited.

2.1 Full pipe bending reference stress solutions

The applied stress (elastically or plastically) used in these equations differs, depending on the way the equations were developed.

2.1.1 Wilkowski and Eiber equation [7]

The Wilkowski and Eiber equation is empirical. The reference stress (σ_{ref}) can be calculated using the following equations:

$$\frac{\sigma_{b,e}}{\sigma_{ref}} = \frac{\eta}{1 - \frac{(1-\eta)}{M_0}} \tag{3}$$

$$M_0 = \sqrt{1 + 0.26\, z + 47\, z^2 - 59\, z^3} \tag{4}$$

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With:

$\eta \quad = 1 - a/t$

$z \quad = {}^{c}/_{\pi R}$

The dimensionless parameter η represents the size of the uncracked ligament (*t-a*) relative to the wall thickness (*t*), whilst the parameter *z* represents the relative length of the crack (*2c*) to the circumference of the pipe ($2\pi R$). The Wilkowski and Eiber equation has been validated based on tests for *z*-values up to 0.15.

2.1.2 Willoughby equation [7]

Another empirical equation is the Willoughby equation which has been experimentally validated for η-values up to 0.2. The Willoughby reference stress is calculated using the next equation:

$$\frac{\sigma_{b,e}}{\sigma_{ref}} = 1 - 1.6\,(1-\eta)\beta$$

With:

$\beta \quad = {}^{c}/_{R}$

The dimensionless parameter β represents the semi-angle of the circumferential crack in a cylinder.

2.1.3 Miller equation [7]

This analytical equation was derived by Miller and is advised by the FITNET procedure. The net section collapse formula contains two different equations depending on the position of the neutral axis:

$$\frac{\sigma_{b,p}}{\sigma_{ref}} = \cos\frac{(1-\eta)\,\beta}{2} - \frac{(1-\eta)\sin\beta}{2} \qquad if\ \beta \leq \frac{\pi}{1+\eta} \tag{5}$$

$$\frac{\sigma_{b,p}}{\sigma_{ref}} = \eta\,\sin\frac{\pi-\beta\,(1-\eta)}{2\,\eta} + \frac{(1-\eta)\sin\beta}{2} \qquad if\ \beta > \frac{\pi}{1+\eta} \tag{6}$$

No limitations have been found for this equation.

2.2 Plate tension reference stress solutions

The applied stress used to calculate the reference stress, is the plastic bending stress ($\sigma_{b,p}$) as defined in equation 2. From a comparison of different plate widths, it was seen that a plate-half width (*W*) of 300mm resulted in the most accurate predictions for the equations described below. Therefore plate half-width (*W*) is assumed to be 300 mm except for these two cases which define the plate width. Firstly the FITNET plate equation, which prescribes a plate width equal to the sum of the crack length (*2c*) and two times the wall thickness (*2t*). Secondly, the net section yielding equation is based on a reference stress calculation using a fixed plate width of 300mm.

2.2.1 Goodall & Webster equation[8]

An analytical reference stress equation for plates subjected to bending and tensile loading was presented by Goodall & Webster. This equation is applicable to *a*/*t*-ratios below 0.5. Reducing this equation to pure tension results in the following reference stress equation:

$$\frac{\sigma_{ref}}{\sigma_{b,p}} = \frac{\gamma + \left\{\gamma^2 + \left[(1-\gamma)^2 + 2\,\gamma\,(\alpha-\gamma)\right]\right\}^{1/2}}{(1-\gamma)^2 + 2\,\gamma\,(\alpha-\gamma)} \tag{7}$$

With:

$\alpha \quad = a/t$

$\gamma \quad = (a\,c)/(Wt)$

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2.2.2 *Lei equation [9]*

The Lei equation is based on a large number of finite element simulations. The crack geometries used to derive the reference stress equation have *a*/*t*-ratios from 0.2 to 0.8 and *a*/*c*-ratios from 0.2 to 1.0. The reference stress, in case of pure tension, can be calculated from the following equations:

$$\frac{\sigma_{b,p}}{\sigma_{ref}} = \frac{d_1}{\gamma+\sqrt{\gamma^2+d_1}} \qquad \psi \leq 1 \tag{8}$$

$$\frac{\sigma_{b,p}}{\sigma_{ref}} = \frac{d_2}{\gamma\,\frac{1-\psi}{\psi-\gamma}+\sqrt{\left(\gamma\,\frac{1-\psi}{\psi-\gamma}\right)^2+\frac{\psi}{\psi-\gamma}\,d_2}} \qquad \psi > 1 \tag{9}$$

With:

$$d_1 = (1-\gamma)^2 + 2\,\gamma\,(\psi-\gamma)$$

$$d_2 = (1-\gamma)\left[2-\psi\,\frac{1-\gamma}{\psi-\gamma}\right] + 2\,\gamma\,(\psi-\gamma)$$

$$\psi = \frac{a}{t}$$

$$\beta_p = \frac{c}{W}$$

$$\gamma = \psi\,\beta_p$$

2.2.3 *FITNET plate equation [3]*

The FITNET plate solution is based on the Lei equation, but prescribes the plate width to be used. The plate width (*2W*) recommended by the FITNET standard is given by:

$$2\,W = 2\,c + 2\,t \tag{10}$$

2.2.4 *Net section yielding equation [10, 11]*

This reference stress equation is basically a flat plate solution, which has been extensively used for the analysis of wide plate test results (with *2W* = 300mm). The reference stress can be calculated using the following equation:

$$\frac{\sigma_{b,p}}{\sigma_{ref}} = 1 - \frac{a\,c}{W\,t} \tag{11}$$

When the defect under consideration has a length exceeding 300 mm, the equation is assumed to be invalid.

2.2.5 *Sattari-Far equation [12]*

This equation has been determined by using finite-element analysis. The analysis was carried out on configurations containing defects with η-values ranging from 0.2 to 1.0 and *c*/*a* ranging from 1 to 5. The reference stress can be calculated solving the following equation:

$$\frac{\sigma_{b,p}}{\sigma_{ref}} = (1-\zeta)^{0.87} \tag{12}$$

With:

$$\zeta = \begin{cases} \dfrac{a\,c}{W\,t} & W < c+t \\ \dfrac{a\,c}{t\,(c+t)} & W > c+t \end{cases}$$

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3 EXPERIMENTAL TEST DATA

The evaluation and comparison of the different equations is based on 59 full scale bend tests on (welded) pipes [5, 6]. The D/t-ratio of the tested pipes ranges from 28 to 90 and c/R is between 0.01 and 0.78. A summary of these test data, including the material's yield strength (σ_{YS}), the bending moment at failure (M_{fail}) and fracture toughness (K_{mat}) is presented in appendix A.

4 ASSESSMENT PROCEDURE

The assessment of each defect requires the calculation of the specific load ratio (L_r) and fracture ratio (K_r) [13]. Because this paper focuses on the comparison of reference stress equations, K_r is always calculated using the widely accepted plate solution derived by Newman and Raju (see Appendix B - [4]). This formula is also used in the BS7910 and the FITNET code [2, 3]. The different reference stress equations discussed in section 2 are used to calculate L_r for every pipe failure.

The failure assessment curve used for this comparison is the curve prescribed in the FITNET code in case of an Option 1 assessment. It should be stated that the FITNET Option 1 does not account for ductile tearing although this was reported for some failures. This can lead to a certain degree of over conservatism resulting from the fact that too much defects are assessed as unsafe. This might be prevented by using a higher assessment level, which unfortunately requires input data that is not available (p.e. the CTOD R-curve and the amount of tearing). Nevertheless, assuming a round-house post-yield behavior, the failure assessment curve for an Option 1 assessment is described by:

$$K_r = f(L_r) = (1 + 0.5\, L_r^2)^{-1/2} \cdot [0.3 + 0.7 \cdot exp(-\mu\, L_r^6)] \qquad L_r \leq 1 \qquad (13)$$

$$K_r = f(L_r) = f(1)\, L_r^{(N-1)/(2\,N)} \qquad 1 < L_r \leq L_{r,max} \qquad (14)$$

With:

$N \quad = 0.3\left(1 - {\sigma_{YS}}/{\sigma_{UTS}}\right)$ [-]

$L_{r,max} \quad = 0.5\left(1 + {\sigma_{UTS}}/{\sigma_{YS}}\right)$ [-]

Combining the calculated failure assessment curve and the failure assessment point (L_r, K_r) enables a plot in the FAD. The FADs for all reference stress equations can be seen in Figure 4 and 5. The assessment points located on the vertical axis represent invalid predictions because η or z was beyond the limitations discussed in section 2.1. For one case, the Newman and Raju equation could not be used because its limitations were exceeded. This assessment point is not presented in the FADs.

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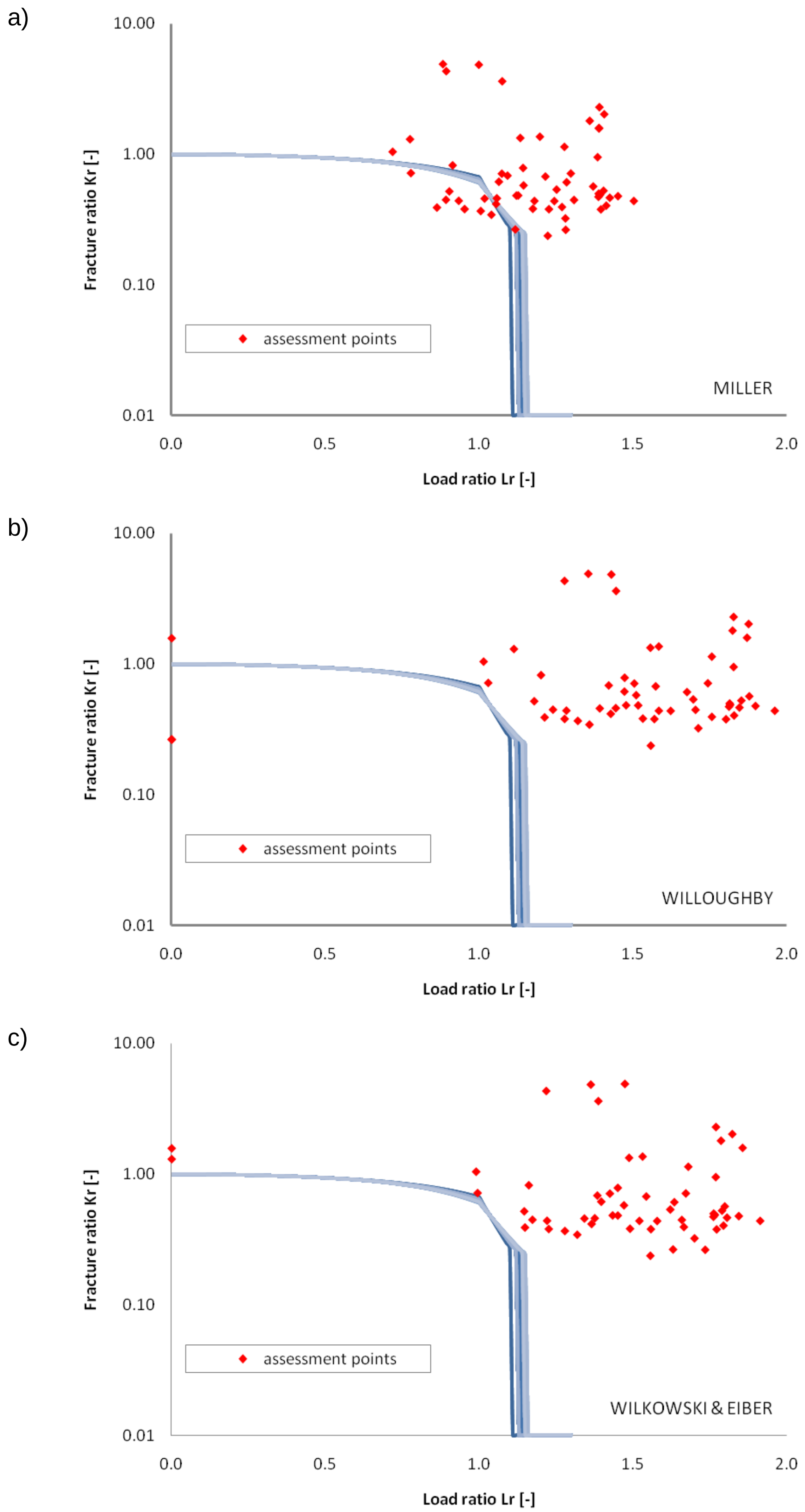

Figure 4. Failure assessment points using the Miller equation (a),
the Willoughby equation (b) and the Wilkowski and Eiber equation (c).

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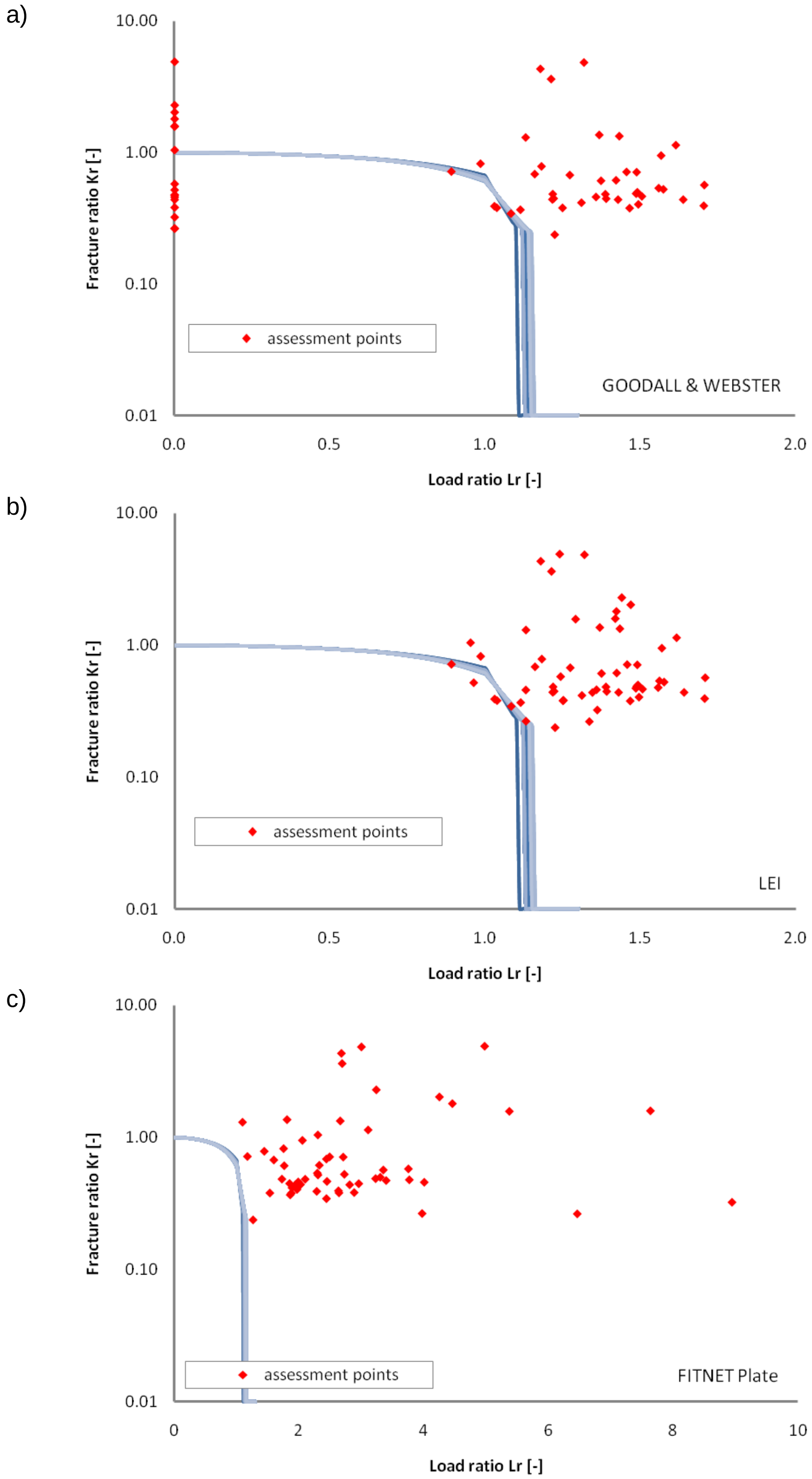

Figure 5. Failure assessment points using the Goodall & Webster equation (a), the Lei equation (b) and the FITNET Plate equation (c).

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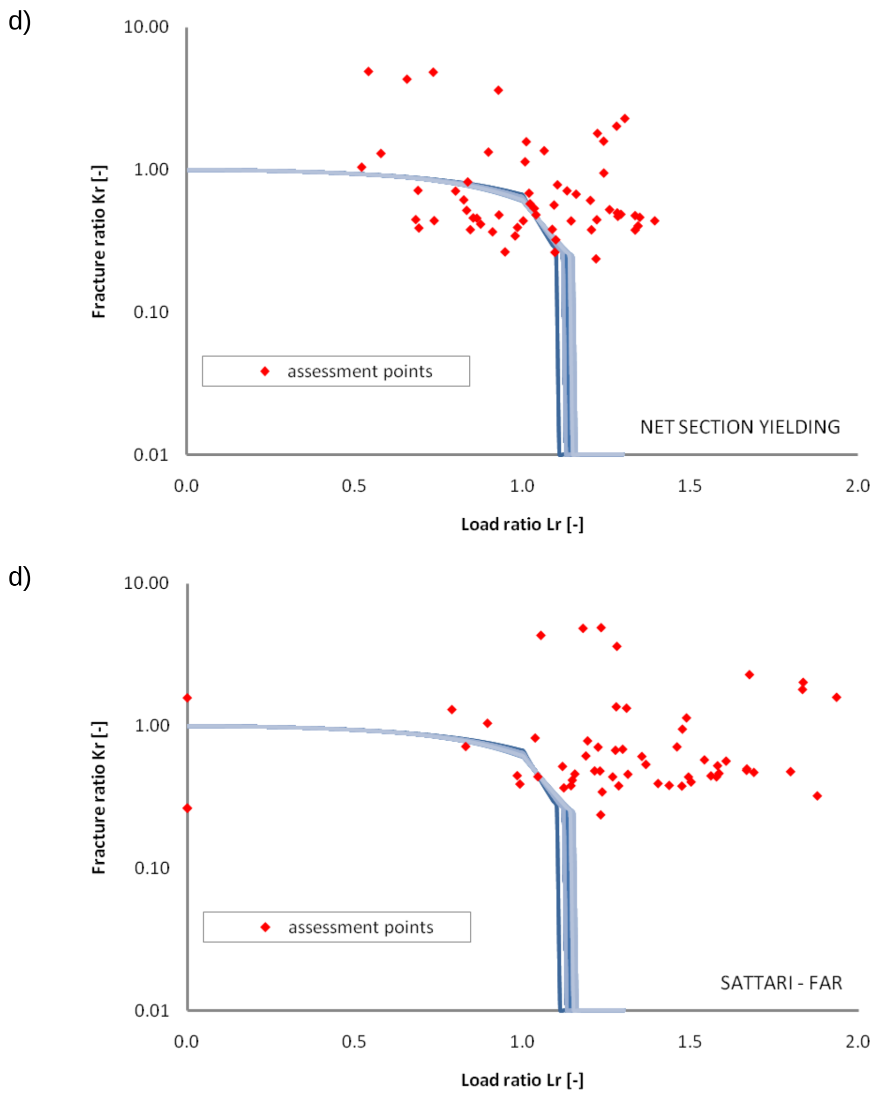

Figure 5. Failure assessment points using the Net Section Yielding equation (d) and the Sattari-Far equation (e).

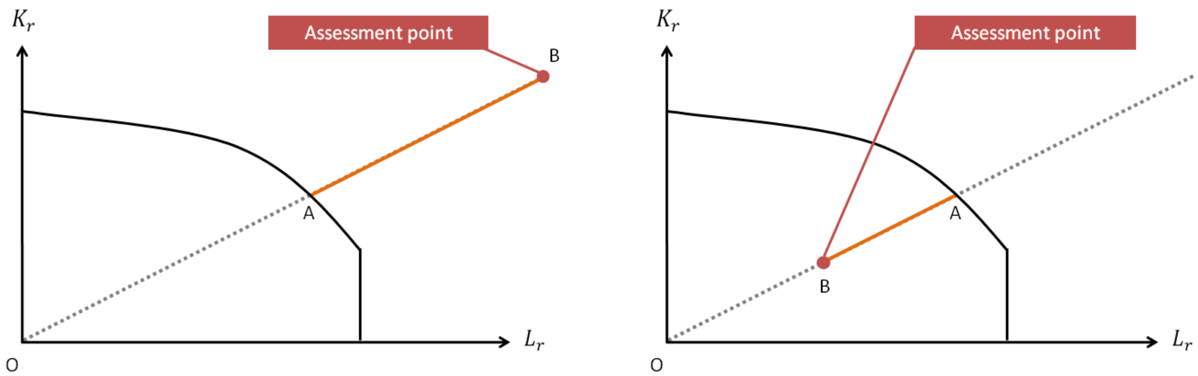

Figure 6. a) Degree of safety of the predicted failures (left) and b) Degree of unsafety of the predicted failures (right)

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5 COMPARISON & DISCUSSION

To judge which of the above reference stress equations predicts failure in the most accurate way, the different FADs have been compared according to different criteria. It should again be mentioned that the fracture ratio has been calculated using the Newman & Raju equation only.

5.1 Number of (un)safe predictions

A first comparison of the different equations is based on the number of unsafe predictions made by each equation (see Figure 7). A prediction is judged unsafe when the assessment point is located under the failure assessment curve because in that case the assessment procedure would have accepted the defect although failure actually occurred.
When it comes to unsafe predictions, the Net Section Yielding equation and the Miller equation show a high number of unsafe predictions. Willing to predict the failure as accurate as possible, this is not preferred. On the other hand, some equations show no unsafe predictions. From this point of view, the use of the Willoughby, Wilkowski & Eiber and the FITNET plate equation is recommended. The fact that the first two equations do not show any unconservative predictions is in good agreement with the empirical way in which they were derived. Noteworthy is also the high number of invalid predictions in case of the Goodall & Webster equation, which appear from deep cracks (deeper than half wall thickness).

5.2 Degree of safety

Besides the number of safe predictions, it is also useful to compare the degree of safety. When the failure is safely predicted, the degree of safety is defined corresponding Figure 6a as:

$$degree\ of\ safety = |AB| \tag{15}$$

The higher this number is, the more conservative the assessment is. From Figure 8 it can be concluded that the FITNET plate reference stress equation should not be recommended although no unconservative predictions were made. The predictions based on this equation incorporate too much conservatism, which does not enable an economical efficient design.
The difference between the other equations is limited, although it can be seen that the Goodall & Webster equation shows the best performance. The difference between this equation and for instance the Wilkowski & Eiber equation is still 18%.

5.3 Degree of unsafety

On the other hand, in case of an unsafe prediction, the degree of unsafety is also important. A small degree of unsafety might be acceptable because other safety factors are included in the assessment procedure. Analogue to the degree of safety, the degree of unsafety is defined as illustrated in Figure 6b:

$$degree\ of\ unsafety = |AB| \tag{16}$$

The degree of unsafety is compared for all reference stress equations in Figure 9. Hereby, the reference stress equations, which showed a large number of unsafe predictions or a large degree of safety are marked in grey and the focus is on the equations left. Taking into account the absence of unsafe predictions in some cases, a degree of unsafety equal to zero can be understood. Focusing on the equations which made unsafe predictions, the Sattari-Far equation and the Lei equation show a larger degree of unsafety than the Goodall & Webster equation.

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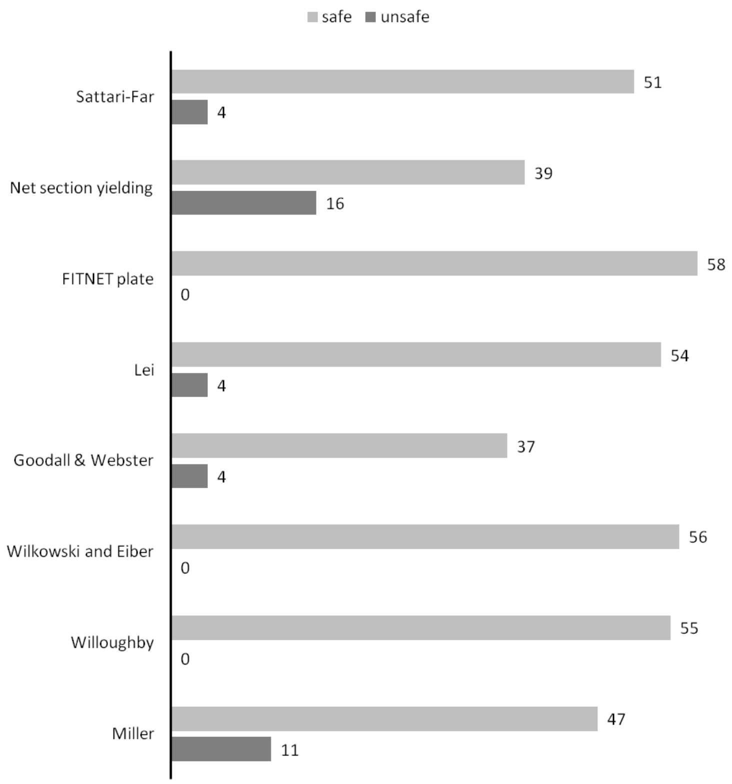

Figure 7. Comparison of the number of (un)safe predictions

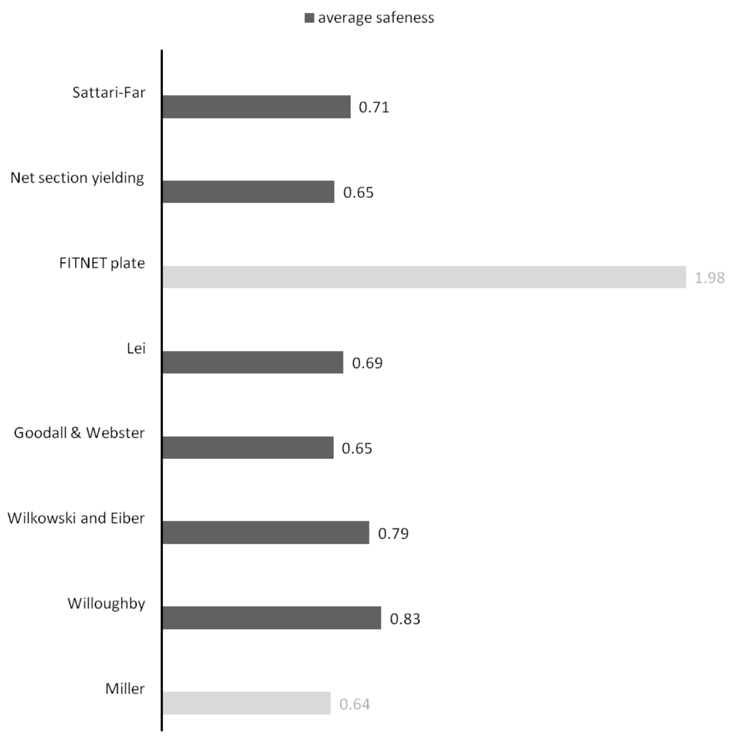

Figure 8. Comparison of the average and maximum safeness

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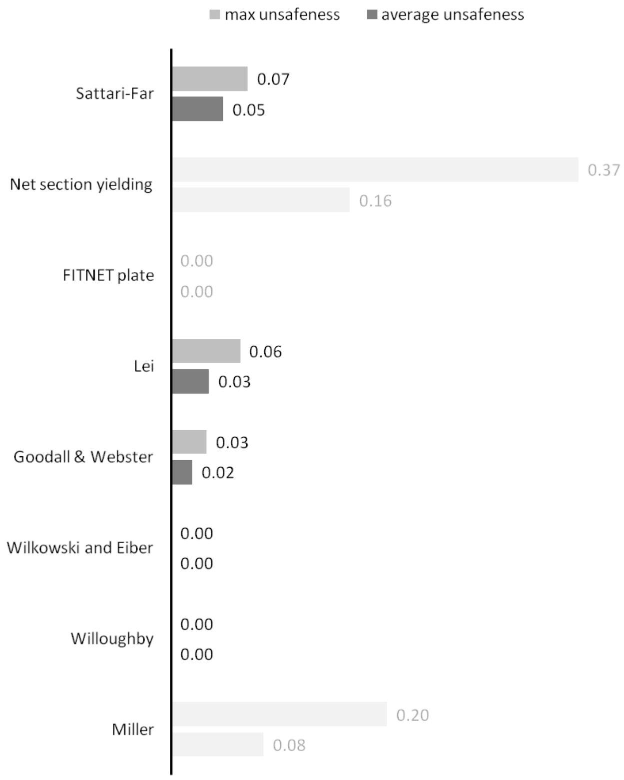

Figure 9. Comparison of the average and maximum unsafeness

6 CONCLUSIONS

Tree criteria have been used to compare and evaluate different reference stress equations. The flat plate solution of Newman & Raju is always used for calculating the fracture ratio. A historical database of full-scale bending experiments on (welded) pipes with circumferential defects has been assessed using the FAD approach. A number of different pipe and plate solutions have been used to calculate the reference stress and load ratio. From this comparison it can be concluded that the use of a flat plate solution, the Goodall & Webster equation, shows a comparable performance to the full pipe solutions by Wilkowski & Eiber and Willoughby. The strength of these three equations, compared to the other equations used in this comparison, is the limited degree of safety and unsafety in combination with a small number of unsafe predictions.

The two empirical equations, namely the Wilkowski & Eiber and the Willoughby equation, have the advantage over the Goodall & Webster equation that no unsafe predictions were made. On the other hand, the degree of safety is larger (+18%) for these empirical equations which might result in a more economical efficient design in case the Goodall & Webster equation is used. The most restrictive condition for using the Goodall & Webster equation is the fact that it is only applicable to shallow defects (a/t < 0.5).

Therefore, we advise to use the Goodall & Webster reference stress equation when an engineering critical assessment has to be performed for circumferential defects in pipes subjected to a bending load. If this equation is not valid, when the defect is too deep, the Wilkowski & Eiber or Willoughby equation should be used.

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7 NOMENCLATURE

σ_{YS}	Yield stress	MPa
σ_{UTS}	Ultimate tensile stress	MPa
σ_{ref}	Reference stress	MPa
L_r	Load ratio	-
K_r	Fracture ratio	-
K_{mat}	Charpy V-notch toughness	MPa√m
ε_f	Strain at failure	-

8 ACKNOWLEDGEMENTS

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9 APPENDIX A: EXPERIMENTAL TEST DATA [5, 6]

D [mm]	t [mm]	2c [mm]	a [mm]	K_{mat} [MPa√m]	σ_{YS} [Mpa]	M_{fail} [kNm]	ε_f [%]
914	11.1	63.5	5.9	136	531	6704	0.5
914	11.1	69.8	5.5	136	531	6704	0.5
914	11.1	68.6	7.8	74	531	5480	0.3
914	11.1	61.0	5.4	74	531	5286	0.3
914	11.1	76.5	10.1	74	531	2504	0.1
914	11.1	81.8	8.8	136	531	6074	0.5
914	11.1	59.3	6.4	74	531	4358	0.2
914	11.1	79.0	9.3	136	531	6074	0.5
914	11.1	63.5	6.3	136	531	7001	0.5
914	11.1	59.6	6.1	136	531	5675	0.4
914	11.1	64.8	5.5	136	531	6732	0.5
914	11.1	60.3	5.5	136	531	6333	0.5
914	10.3	300.0	4.1	154	689	5926	0.8
914	10.3	300.0	3.6	154	689	5926	0.7
914	11.1	265.0	3.3	136	531	5276	0.3
914	11.1	278.0	3.2	136	531	5888	0.4
914	11.1	279.0	3.9	127	466	3811	0.2
914	11.1	331.0	3.7	127	466	3616	0.2
914	11.1	75.0	3.5	127	466	6074	0.5
1067	15.0	14.0	0.9	131	496	10349	0.7
1067	15.0	38.0	3.0	131	496	10349	0.8
1067	15.0	70.0	8.0	131	496	10349	0.6
914	11.1	315.0	3.7	187	441	4887	0.3
914	11.1	282.0	3.1	127	466	4358	0.3
914	11.7	280.0	2.9	128	470	5219	0.4
914	11.7	134.0	3.7	128	470	4514	0.3
914	11.7	116.0	2.2	128	470	6423	0.7
610	6.8	100.0	3.1	121	532	1225	0.2
610	6.8	199.0	2.8	121	532	1074	0.2
610	6.8	51.0	3.1	121	532	1363	0.3
610	6.8	107.0	3.9	120	532	1290	0.2
508	8.7	108.0	8.7	20	469	1306	-
508	8.7	42.6	4.8	20	469	1437	-
508	8.7	44.9	8.7	20	469	1396	-
508	8.7	52.4	6.4	20	469	1383	-
508	8.7	52.4	6.8	20	469	1410	-
508	8.7	50.0	5.9	20	469	1437	-
762	15.8	239.0	11.9	20	573	4064	-
762	15.8	239.0	7.9	20	573	4312	-
762	15.8	119.0	7.9	20	573	5422	-
762	15.8	239.0	7.9	20	573	4826	-
914	11.7	112.0	2.0	126	460	6207	0.8
914	11.7	141.0	3.9	126	460	6148	0.4
914	11.7	300.0	3.5	126	460	4516	0.3
762	19.0	105.0	3.5	81	472	6244	0.4
762	19.0	139.0	3.7	99	472	6564	0.5
762	19.0	125.0	5.0	121	472	5737	0.5
711	25.4	127.0	10.9	114	470	7524	0.6
762	15.9	89.0	6.4	58	487	4007	0.2
762	15.9	597.0	3.1	58	487	3231	0.2
762	15.9	89.0	2.5	58	487	5087	0.9
914	25.0	178.0	4.6	188	486	14974	1.1
914	25.0	199.0	5.9	93	499	14056	0.6
914	25.0	212.0	6.1	73	514	8103	0.2
914	25.0	207.0	6.1	60	526	12782	0.5
914	25.0	191.0	12.3	188	486	13178	0.6
914	25.0	205.0	12.0	93	499	12573	0.5
914	25.0	210.0	11.5	73	514	11508	0.3
914	25.0	210.0	15.4	60	526	7310	0.2

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10 APPENDIX B: NEWMAN AND RAJU EQUATION

$$K_I = (S_t + H\, S_b)\sqrt{\pi \frac{a}{Q}}\, F$$

With:

$$Q = 1 + 1.464 \left(\frac{a}{c}\right)^{1.65}$$

$$F = \left[M_1 + M_2 \left(\frac{a}{t}\right)^2 + M_3 \left(\frac{a}{t}\right)^4\right] f_\phi\, g\, f_w$$

$$M_1 = 1.13 - 0.09 \frac{a}{c}$$

$$M_2 = -0.54 + \frac{0.89}{0.2 + {}^{a}/_{c}}$$

$$M_3 = 0.5 - \frac{1.0}{0.65 + {}^{a}/_{c}} + 14 \left(1.0 - \frac{a}{c}\right)^{24}$$

$$g = 1 + \left[0.1 + 0.35 \left(\frac{a}{t}\right)^2\right] (1 - \sin\phi)^2$$

$$f_\phi = \left[\left(\frac{a}{c}\right)^2 \cos^2\phi + \sin^2\phi\right]^{1/4}$$

$$f_w = \left[sec \frac{\pi\, c}{2\, W}\sqrt{\frac{a}{t}}\right]^{1/2}$$

$$H = H_1 + (H_2 - H_1) \sin^{p}\phi$$

$$p = 0.2 + \frac{a}{c} + 0.6 \frac{a}{t}$$

$$H_1 = 1 - 0.34 \frac{a}{t} - 0.11 \frac{a}{c} \frac{a}{t}$$

$$H_2 = 1 + G_1 \frac{a}{t} + G_2 \left(\frac{a}{t}\right)^2$$

$$G_1 = -1.22 - 0.12 \frac{a}{c}$$

$$G_2 = 0.55 - 1.05 \left(\frac{a}{c}\right)^{0.75} + 0.47 \left(\frac{a}{c}\right)^{1.5}$$

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With:

h Half-length of cracked plate [mm]

M Applied bending moment [Nm]

S_b Remote bending stress on outer fiber [Pa]

$$= \frac{3\,M}{W\,t^2}$$

S_t Remote uniform tension stress [Pa]

t Plate thickness [mm]

ϕ Parametric angle of the ellipse [deg]

Boundary condition:

$$0 < \frac{\mathrm{a}}{\mathrm{c}} \leq 1$$

$$0 \leq \mathrm{a/t} < 1$$

$$c/W < 0{,}5$$

$$0 \leq \phi \leq \pi$$

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