

# SYMMETRY AND SIMILARITY PROPERTIES OF 2D FIGURES

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**Abstract** A new algorithm has been developed to sort two-dimensional figures by their shape. The algorithm can find the figures having equal or approximately equal shape independently from geometrical measures and the complexity of the shapes. It is based on the definition of the symmetry-diagram which was introduced for symmetry recognition originally. The new method can be used for sorting different parts at assembly lines and for reverse engineering, too.

**Keywords:** Symmetry-diagram, symmetry detection, shape detection

## 1 INTRODUCTION

The sorting algorithm takes advantage of the feature of the symmetry-diagram (in other name the shape-diagram) that it shows a unique shape property of the two-dimensional (2D) figures independently from geometrical measures i.e. it is a shape factor.

The symmetry-diagram was introduced for symmetry detection originally [1]. The problem of symmetry detection has been extensively studied in numerous fields including visual perception, computer vision, robotics, computational geometry and reverse engineering. Early methods concentrated on finding perfect symmetries in 2D or 3D point sets [2], [3]. Since the restriction to exact symmetries limits the use of these methods for real-world objects, a method was introduced for computing approximate global symmetries in 3D point sets [4], but the complexity of the algorithm makes it impractical for large data sets. The notion of approximate symmetry was formalized by expressing symmetry as a continuous feature [5]. The examination of the correlation of the Gaussian image was proposed to recover global reflective and rotational symmetries [6]. A shape descriptor was introduced that concisely encodes global reflective and rotational symmetries [7], [8]. Different applications based on generalized complex moments [9], grouping feature points [10], [11], [12], isometric transformations [13], planar reflective symmetry transform [14] and generalized symmetry transformation [15], [16] are used in image processing and mesh processing for detecting exact local and global reflection-symmetry and rotation- symmetry of 2D and 3D images.

The symmetries are often not exactly present, but only approximately present, due to measurement errors in the scanning process, and approximation and numerical errors in model reconstruction during reverse engineering [17]. Beside this, different CAD systems often use different tolerances [18], and what is symmetric in one CAD system may be only approximately symmetric in another. To solve these problems new algorithms based on the B-rep model were developed to find approximate rotational and translational symmetries of 3D forms built from simple geometric units [19] and complex 3D forms [20] in reverse engineering.

However a number of outstanding methods for symmetry recognition can be found in the literature, most of them do not take advantage of the fact that every symmetry axis of a 2D figure has to cross the gravity centre [2-20].

The symmetry-diagram is defined as the followings [1]:

- The contour of the 2D figure is needed to be known for the definition.
- A number between 0 and 1 signed by 'Z' ( $0 < Z \leq 1$ ) is computed for any line crossing the gravity centre. This 'Z' parameter, in other name the symmetry parameter is defined so that it equals to 1 if the line is an exact symmetry axis and it approximates to 1 if the line is an approximate symmetry axis. The less the line can be considered as a symmetry axis the less is the value of 'Z'.
- The symmetry-diagram shows the values of 'Z' for a set of lines revolving around the gravity centre in an angle range of  $180^\circ$ .

Independently from the fact that a given 2D figure is symmetric or not the symmetry-diagram is a shape factor: The symmetry-diagrams are different for the figures having different shape. Beside this: the symmetry-diagrams of coincident figures are the same and the symmetry-diagrams of approximately coincident figures are approximately same. The symmetry-diagrams of geometrically similar figures are the same (independently from geometrical measures) and the symmetry-diagrams of approximately similar figures are approximately same. Because of this important property of the symmetry-diagram it can be applied for recognition of figures having the same or approximately same shape (independently from geometrical measures). There is only one limit of the recognition: figures having more closed concentric curves as boundary (e.g. two concentric circles, the spacer) have the same symmetry-diagram.

## 2 THE SORTING ALGORITHM

Effective sorting and seeking (e. g. faulty seeking) algorithms are applied in the most different fields as for example in the data sorting [21], [22], in the sound technique [23] and in the image technique [24-33]. In the image technique one of the most relevant problems is the shape detection consisting of two different partial problems: the shape detection as boundary detection of the image segmentation and the shape detection (shape recognition) as making differences between figures having different shape. In the case of the boundary detection a lot of effective algorithms are known [24-28]. In the field of the shape recognition the solutions of the most different special problems are worked out as for example a fruit sorting algorithm [29], a face detection algorithm [30], a traffic sign shape classification algorithm [31]. A so called distance measure [32] and the gradient-based shape descriptor [33] were developed to distinguish different classes of the 2D figures.

The new sorting algorithm shown in this work is a general algorithm which can be used without limits in all 2D application fields of the shape recognition.

Knowing the definition of the symmetry-diagram the sorting algorithm consists of several simple steps:

- The input of the algorithm is the set of the boundaries of the 2D figures.
- The symmetry-diagram of every 2D figure is computed.
- The subsets of same or approximately same diagrams are divided.

We show the process in a case-study from the mechanical engineering. In Figure 1 a photo of screws, screw-nuts and spacers having different measures can be seen. The photo shows the result of an image process, as well. The red curves are fitted to the boundaries of the elements and the set of these contours is the input of the sorting algorithm.



Figure 1. The input of the sorting algorithm is the set of the contours of the elements signed by red curves. The numbers are the identifications of the elements.

At using the sorting algorithm it has to be taken down the outer outline of configurations. Part aggregation can be seen in Figure 1. where the red colour marks the outlines of parts. Numbers mark certain parts in the sake for later identification. We handle separately with the algorithm the outline of certain parts. In case of each part We take down the picture points of outlines forming the parts in a co-ordinate system. The symmetry-diagram calculated to each part after the algorithm run can be seen in Figure 2. arranged. It can be seen well on the symmetry-diagrams the parts examined can be simply grouped. The sorting of parts can be carried out by taken to the Z-parameter to difference but characteristic ranges.

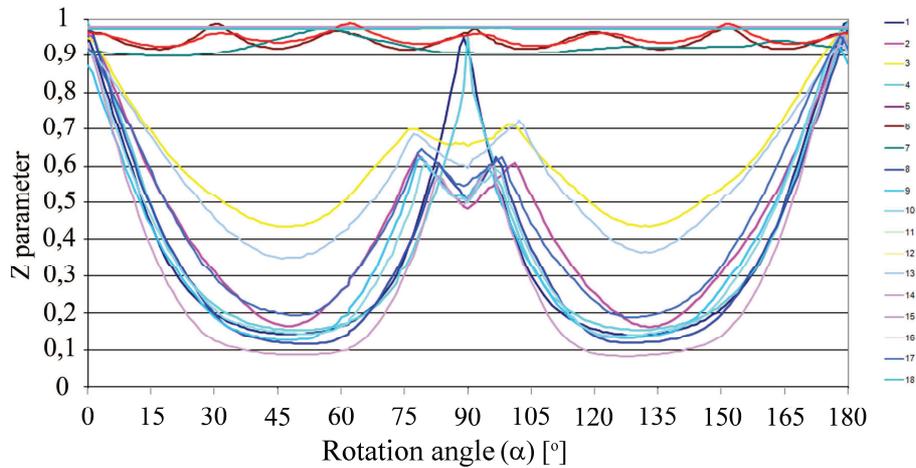


Figure 2. The summed-up results

Those parts are similar to circle which Z parameter is greater than 0,9 in all rotation angle-positions of symmetry-diagrams. For each diagram to be seen in Figure 3. is true  $0,9 < Z < 1$ . The great majority of curves can be found within a strict range in the Figure, these can be seen magnified in Figure 4. The curve marked 7 in Figure 3. though is circular but it differs significantly from the others therefore it has to be examined separately. The curve also shows high Z parameter on the whole however there is also a preferred direction where the value of symmetry-parameter  $Z=0,976$  at  $\alpha = 61^\circ$ . That means such axis can be taken down surely to the configuration examined which approximate symmetry.

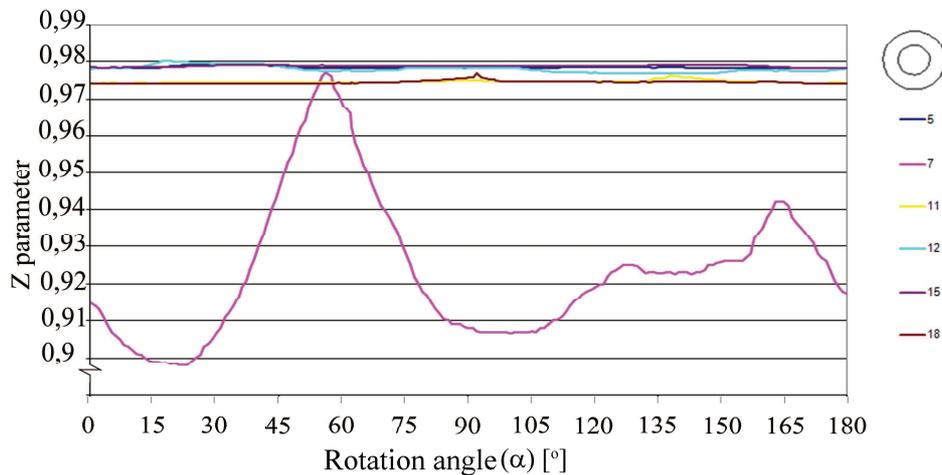


Figure 3. Symmetry-diagrams of circular configurations

The number 7 part is put together by a set of washers and pipe-piece with the marks of Figure 2. The configuration stuck together during taking a photo succeeded to photograph on a optional position this causes deformation on the picture examined. In this case just this deformation makes possible to recognize it as a faulty part. The algorithm would have been surely recognized as a washer if it had been arranged exactly in the vertical position under the camera. Avoiding such cases it is worth to examine pictures made from camera line-up with two different angular positions.

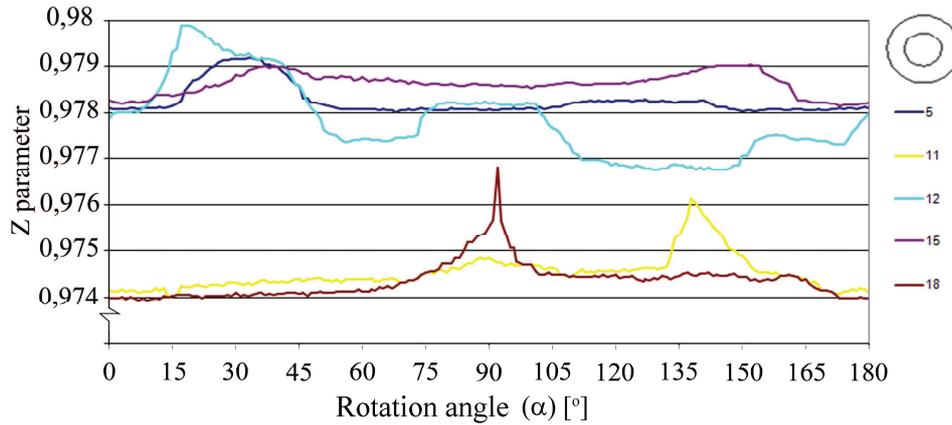


Figure 4. A strict range of circular configuration

Further grouping possibility presents oneself in Figure 4. The value of Z parameter can be found between 0,974-0,98 in 0,006 zone. Examining the shape of curves found in the very strict zone it comes to light that the 11. and 18. configurations differ in some measures from the other similar circular configurations. Beyond on that they resemble to the circle there is surely on then a preferred point (the shot of spring washer). The outstanding parameter values to be found on the various angular positions of the 11. and 18. configuration curves derive from the different orientations of washers.

The number 5. part of the Figure 22. is also a spring washer which can not be distinguished with the present single-camera examination from the other washers. In this case the examination from two separate camera line-ups gives sure solution.

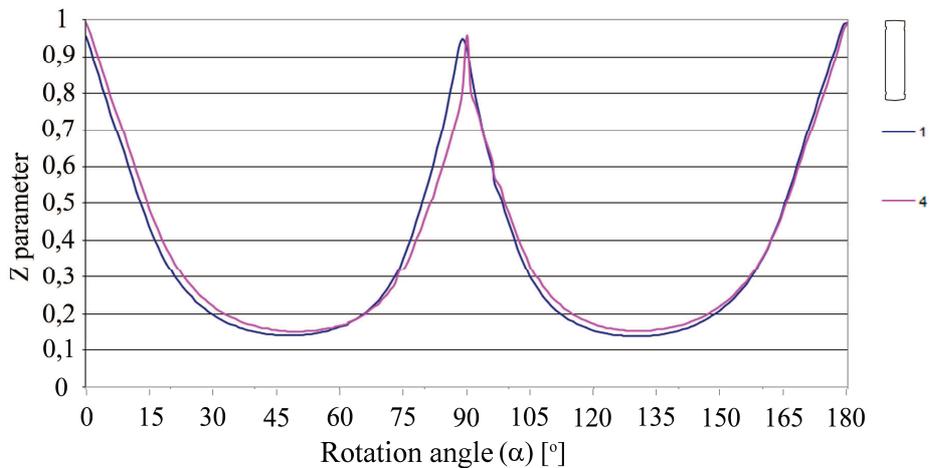


Figure 5. Rectangular like configurations with large side-edge ratio

The characteristic of curves to be seen in Figure 5. is that it can be found parameter near to  $Z=1$  in two angular positions. This type symmetry-diagram is characteristic to rectangulars with at least 1/6 side ratio. On the diagrams of number 1. and 4. pin like parts can be seen in present case. In case of both parts the value of visible approximate symmetry in the  $90^0$  angular position of the curves is smaller than in  $0^0$  angular position of these. The non full symmetry is due to smaller slots on configurations.

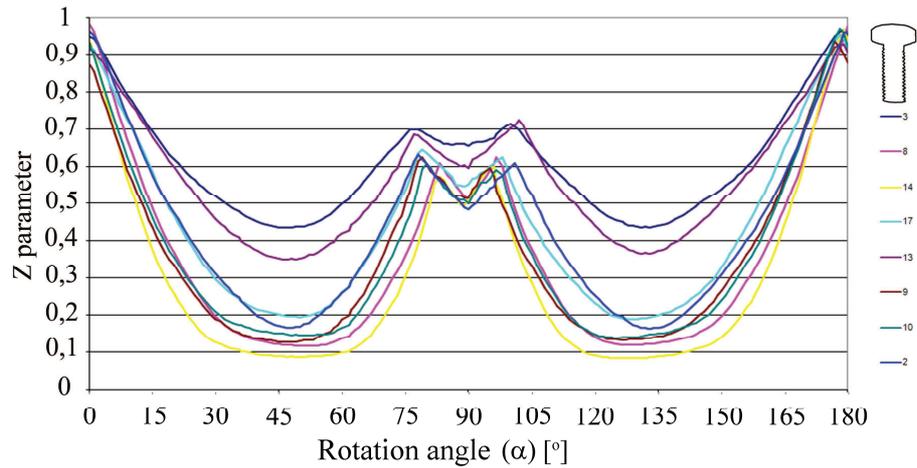


Figure 6. Curves of bolt like configurations

The curves of Figure 6 resemble to curves shown in Figure 5. here it is also true that they mark rectangular like configuration with large side-edge ratio (in 2D rectangular, in 3D it can be just like cylindric). The cause of local parameter reduction of the symmetry-diagram in  $90^0$  angular position is the result of irreversible symmetry defect. The symmetry deformation derives from the head formation of bolts and rivets in present case. The aggregation contains also bolts and rivets during examination to which We have carried out the run. The algorithm can not distinguish these parts in this resolution. If there are previous knowledge concerning the configuration examined then increasing the resolution of picture processing these parts already can be distinguished, too.

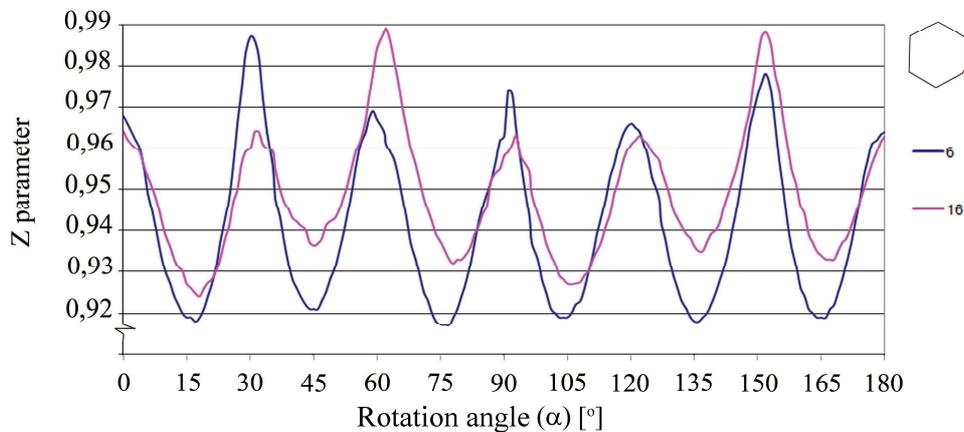


Figure 7. Symmetry-diagram characterizing hexagon parts

The curves having got 6 local maximums can be seen in Figure 7. this means that the configuration 6. and 16. are hexagon among the configurations examined. That comes also to light from the symmetry-diagram that damages, deformations can be found on the hexagon configurations examined. The curves should reach six times the  $Z=1$  value in ideal case.

It can be seen from the results got during the run that the software produced by us recognizes the similarity of optional planar configurations. The algorithm is not suitable determining symmetry characteristic in case of configurations with concentric outline (for example: washer). This during sorting of parts it can not neither distinguish the washer from a disc.

### 3 CONCLUSIONS

The symmetry searching algorithm developed during our research work is suitable to sort for example bulk joint-elements. Presently the algorithm makes possible the classification according to shape. The pre-sorting of bulk parts can be solved by this. It can not be used to sort according to dimension. However the algorithm determines the symmetry characteristics based on digital pictures by setting up proportions from geometric dimensions, because of from the pictures examined the geometric dimensions of configurations can be also decided during processing. By utilizing this the algorithm can be made suitable to classify according to dimension, too.

It has been started the realization in technological circumstances the automatized pre-sorting of bulk joint-elements. We have started with the engineers of the company renewing aeroplane power plants in Veresegyház the practical realization of the method worked out by me. The work is presently in the phase of planning and instrument obtaining.

The cold forming standard (MSZ EN 10142:2000) containing low-carbon content strip steel and plate hot-dip galvanized qualifies the determination the so called zinc-flowers formed during the process by measuring number that can be determined with non exact mode. The standard uses the following ideas for putting in row according to dimension of zinc-flowers formed on the surface galvanized: small-, medium-, large zinc-flowers. Putting in row the zinc-flowers according to the standard is important concerning use, as for example cold formability, colorability.

I think so that with the symmetry searching method worked out by us, based on the symmetry characteristics it could be determined unambiguously the dimension steps of zinc-flowers according to various fields of use.

There are in trade straw cutters developed for this aim with various constructions for using as an energetics aim of agricultural crops. The most important viewpoint is the sturdy structure durability and the reliability at designing harvesting machines. The size and the shape of chaff are an important viewpoint during using for energetics aim. The dimension, shape and surface of the chippings have got an emphasized rule concerning the firing technique. The presently wide-spread stem chippings is suitable for firing, but for operating boilers with good efficiency is indispensable to produce homogeneous-and optimum size solid fuel for equipment given.

By using the method drawn up to search the symmetry it can be worked out an appliance provided with an online data collecting system which qualifies the chippings considering morphology during cutting chaff. Based on the results by this way a control can be developed for optimizing the technological parameters of straw cutters.

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