

STRESS UPDATE ALGORITHM FOR NON-ASSOCIATED FLOW METAL PLASTICITY

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Keywords: material constitutive model, associated flow rule, non-associated, yield function.

Abstract. In this paper we present the continuum plasticity model based on non-Associated Flow Rule (non-AFR) for Hill's48 quadratic yield function. In case of non-AFR, Hill's quadratic function used as plastic potential function, makes use of plastic strain ratios to determine the direction of effective plastic strain rate. In addition, the yield function uses direction dependent yield stress data. Therefore more accurate predictions are expected in terms of both yield stress and strain ratios at different orientations. We implemented a modified version of the non-associative flow rule originally developed by Stoughton [1] into the commercial finite element code ABAQUS by means of a user material subroutine UMAT. The main algorithm developed includes combined effects of isotropic and kinematic hardening [2]. This paper assumes proportional loading cases and therefore only isotropic hardening effect is considered. In our model the incremental change of plastic strain rate tensor is not equal to the incremental change of the compliance factor. The validity of the model is demonstrated by comparing stresses and strain ratios obtained from finite element simulations with experimentally determined values for deep drawing steel DC06. A critical comparison is made between numerical results obtained from AFR and non-AFR based models.

1. INTRODUCTION

Associative flow rule (AFR) as the dominant theory in metal plasticity is known to use an identical formulation for both the yield and the plastic potential functions. Different yield criteria have been developed which employ a series of parameters in their formulation based on either strain ratios or directional yield stresses or a combination of these two. In case of plastic deformation, the plastic strain increment is found by the plastic potential function which is equal to the yield stress function. Thus, using a plastic potential function to address zero dilatancy (i.e. zero or negligible volume change after plastic deformation), automatically implies using the same pressure insensitive function as yield stress function (normality condition). This is why an associated flow rule is not capable of modeling both zero dilatancy and pressure sensitive yield stresses. However, uniaxial tension and compression tests on iron based metals and aluminum, reported by Spitzig and Richmond [3], revealed the (linear) dependency of yield stress on hydrostatic pressure. This observation, together with the negligible plastic dilatancy assumption for metals, opened a discussion about the validity of associated plasticity. Attention has been focused on non-associated flow rules (non-AFR), thus describing the plastic flow independently from the yield stress function in constitutive models of metals. This work is based on a non-AFR originally developed by Stoughton [1], which in turn is a typical version of a more generalized and pressure sensitive one developed by Stoughton and Yoon [4].

2. NUMERICAL IMPLEMENTATION

At room temperature plastic deformation happens due to dislocation glide on given crystallographic planes resulting in microscopic shear deformations [5]. This motion results in an accumulation of microstructural obstacles, thus increasing the slip resistance (hardening) [6]. Considering only proportional loading, this resistance of a metal against further plastic deformation is described by an isotropic hardening function in which the yield surface of the metal expands radially. In metal forming applications, the renowned Swift isotropic hardening model is commonly used for non-saturating materials such as steels. However, in some cases the Swift model is unable to perfectly fit to experimental data. Therefore we suggest an isotropic hardening function for DC06, which is a modified version of Swift's law (Eq. 1). This equation is able to accurately model the specific non-linearity observed before 0.1 amount of true plastic strain, what is not feasible with the ordinary Swift law (Fig. 1).

The Swift hardening law is described as

$$\sigma^{iso} = C_R(\epsilon_0 + \epsilon^p)^n \quad (1)$$

The proposed modification to Swift's law is

$$\sigma^{iso} = C_R(\epsilon_0 + \epsilon^p)^n - C_R'(\epsilon_0 + \epsilon^p)^{n'} \quad (2)$$

Material properties for DC06 are described in Table 1.

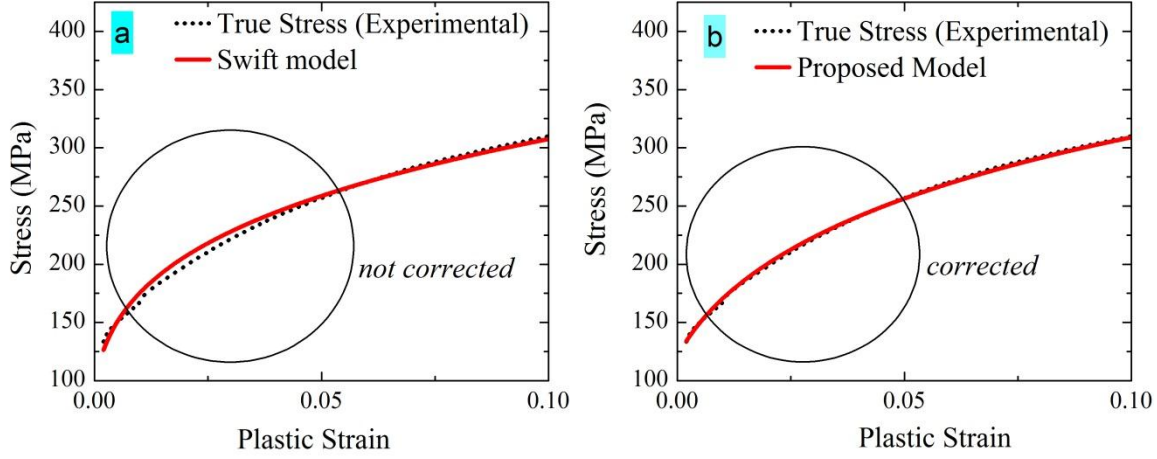


Figure 1. Hardening curve for DC06 at rolling direction, a) Swift law, b) modified Swift law.

Table 1. Material parameters for Swift and modified Swift hardening law

Swift law			Modified Swift law					
C_R [MPa]	ϵ_0	n	C_R [MPa]	ϵ_0	n	C_R' [MPa]	ϵ_0	n'
551.004	0.00076	0.251	447.314	0.0014	0.8104	890.291	0.00518	0.3767

To describe the orthotropic behaviour of the deep drawing steel, the classical Hill'48 continuously differentiable yield stress function (f_y) has been used. The yield criterion F is described by

$$F = f_y - \sigma^{iso} = (\sigma_{11}^2 + \lambda_y \sigma_{22}^2 - 2\nu_y \sigma_{11}\sigma_{22} + \rho_y \sigma_{12}^2)^{1/2} - \sigma^{iso} \quad (3)$$

where ν_y , λ_y and ρ_y are yield stress function parameters defined as

$$\lambda_y = (\sigma_0 / \sigma_{90})^2, \quad \nu_y = (1 + (\sigma_0 / \sigma_{90})^2 - (\sigma_0 / \sigma_b)^2) \quad \text{and} \quad \rho_y = 2(\sigma_0 / \sigma_{45})^2 - 0.5(\sigma_0 / \sigma_b)^2 \quad (4)$$

The stress components σ_0 , σ_{45} and σ_{90} are respectively the yield stress at RD (rolling direction), DD (diagonal direction) and TD (transverse direction) obtained from uniaxial tensile tests. Based on the work of Butuc et al [7], the equibiaxial yield stress (σ_b) is calculated from $\sigma_b / \sigma_0 = 1.045$. If the exact formulation of f_y is used for determining the direction of plastic strain rate, then the common associated flow rule is obtained. One of the drawbacks of using AFR based on Hill'48 (using f_y in Eq. 3) could be the rough prediction of strain ratios. This is because the material parameters in Eq. 4 are calibrated based on direction dependent yield stresses. One may suggest to use a strain ratio dependent yield function to overcome the poor prediction of strain ratios. In this case, strain ratios would be predicted accurately due to the strain ratio based calibration approach. However, a poor prediction of yield stress results at different orientations can be obtained. Some researchers suggest an optimized approach and calibrate material parameters based on both strain ratios and direction dependent yield stresses resulting in a better prediction compared to previous approaches [8].

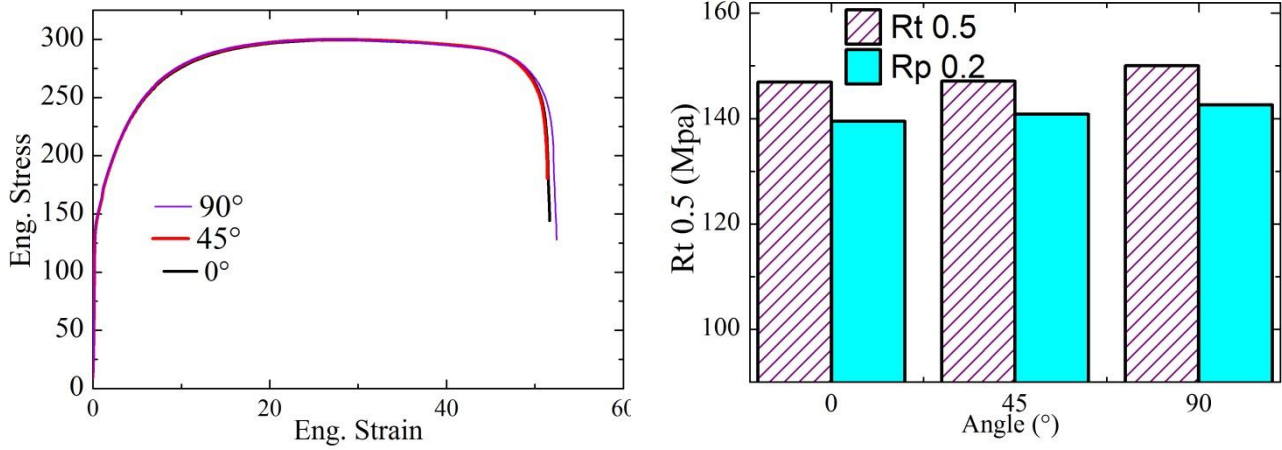


Figure 2. Experimental tensile test diagrams for DC06 (0.8mm thickness) at RD, DD and TD (left diagram). Yield stress at 0.2% plastic strain and 0.5% total strain (right diagram).

The experimental tensile test results and yield stresses at 0.2% plastic strain and 0.5% total strain for deep drawing steel DC06 (sheet thickness 0.8mm) are plotted in Fig. 2. Judging from the stress-strain diagrams for different orientations we may conclude that DC06 possesses isotropic behaviour. But as will be shown later (Fig. 3), the plastic strain ratios of this material reveal an anisotropic plastic behaviour. Therefore describing this behaviour using Hill'48 in a framework based on AFR seems impossible. However, using a plastic potential function calibrated from strain ratios to control the plastic behaviour of the metal may improve the results of the constitutive model. This could explain that non-AFR based models may be capable of solving the so-called anomalous behaviour in materials with normal anisotropy less than one.

In this case (non-AFR) the Hill's potential function (f_p) is described by

$$f_p = \sigma^{iso} = (\sigma_{11}^2 + \lambda_p \sigma_{22}^2 - 2\nu_p \sigma_{11}\sigma_{22} + \rho_p \sigma_{12}^2)^{1/2} \quad (5)$$

where ν_p , λ_p and ρ_p are plastic potential function parameters

$$\lambda_p = r_0(1 + r_{90}) / (r_{90}(1 + r_0)), \quad \nu_p = r_0 / (1 + r_0) \quad \text{and} \quad (6)$$

$$\rho_p = (r_{90} + r_0)(1 + 2r_{45}) / (2r_{90}(1 + r_0))$$

The plastic strain increment is obtained by:

$$d\varepsilon^p = d\lambda n \quad (7)$$

where n is the first derivative of the plastic potential function

$$n = \partial f_p / \partial \sigma \quad (8)$$

and $d\lambda$ is the plastic compliance factor that determines the magnitude of effective plastic strain increment.

The amount of plastic work calculated from the yield function is equal to that calculated based on Cauchy stress. This is called the principle of plastic work equivalence, and is described by

$$f_y d\varepsilon^p = \sigma d\varepsilon^p \quad (9)$$

where ε^p is effective plastic strain.

Applying Euler's theorem for a first order homogenous function, here the plastic potential function, results in

$$\sigma (\partial f_p / \partial \sigma) = f_p \quad (10)$$

Substituting Eq. 7 in Eq. 9 and then applying Euler's theorem Eq. 10 gives

$$d\varepsilon^p = \sigma d\varepsilon^p / f_y = d\lambda (\sigma (\partial f_p / \partial \sigma)) / f_y = d\lambda f_p / f_y \quad (11)$$

This equation relates the increment of equivalent plastic strain to the plastic compliance factor. Implementing this equation into an implicit stress update algorithm results in very laborious calculations especially when kinematic hardening effects are involved. For this reason some researchers assume equality of equivalent plastic strain increment and compliance factor [9,10]. Similar to Cvitanic et al [11] we take the non-simplified approach as described by Eq.11 into account for the integration algorithm. Another crucial constraint in both AFR and non-AFR is the consistency condition which keeps the stress state of the metal on the yield surface in case of plastic deformation:

$$m d\sigma = H d\varepsilon^p \quad (12)$$

where H is the derivative of the isotropic hardening function with respect to effective plastic strain.

Commercial finite element codes are displacement driven and the stress tensor is found by discretising the strain tensor into elastic and plastic strain tensors [12].

$$d\sigma = C^e(d\varepsilon - d\varepsilon^p) \quad (13)$$

Determining plastic and elastic strain tensors can be done by different integration algorithms from which the fully implicit one gained most attention due to its unconditional stability which is lacking in an explicit approach. In the stress update algorithm based on implicit integration, the trial stress function (Eq. 14) updates the stress tensor outside the yield surface. In the next step the stress tensor is brought back to the yield stress by means of a plastic corrector.

$$\sigma^{tr} = \sigma_n + \Delta \varepsilon C^e \quad (14)$$

where σ^{tr} and σ_n are trial stress and stress at the last converged step, respectively.

This procedure is performed by solving a series of non-linear equations by means of Newton-Raphson iterations to iteratively bring back the stress state onto the yield surface at the current time step. This method requires defining a series of residual functions for plastic strain, Cauchy stress and yield function as described in Eq. 15, 16 and 17.

$$r^\sigma = \sigma - \sigma_n - C^e(d\varepsilon - d\lambda n) \quad (15)$$

$$r^p = \Delta \varepsilon^p f_y - \Delta \lambda f_p \quad (16)$$

$$r^F = f_y - \sigma^{iso} [\varepsilon^p_n + d\varepsilon^p] \quad (17)$$

Using Taylor's truncated series for Eq.15-17, after extensive manipulations we reach to

$$d\Delta\sigma = -\Theta^{-1} r^\sigma - \Theta^{-1} d\Delta\lambda C^e n, \text{ where } (\Theta = I + \Delta\lambda C^e (\partial f_p / \partial \sigma)) \quad (18)$$

$$d\Delta\varepsilon^p = (df_y \Delta\lambda + d\Delta\varepsilon^p f_p + r^F \Delta\varepsilon^p - r^p) / (f_y + \Delta\varepsilon^p H) \quad (19)$$

where $d\Delta$ denotes to incremental change in the constitutive variable. In Eqs. 18 and 19 $d\Delta\lambda$ is described as

$$d\Delta\lambda = (r^F - B_1 \Theta^{-1} r^\sigma + H r^p / f_y) / (H f_p / f_y + \Theta^{-1} C^e n) \quad (20)$$

Using the developed implicit material subroutine each increment starts with checking the yield condition (Eq.3) in which the trial stress is employed from Eq. (14). If the requirement of plastic flow is fulfilled then residual functions for stress, yield function and effective plastic strain are obtained from Eqs. 15-17, respectively. Subsequently the iterative change of compliance factor ($d\Delta\lambda$) can be calculated by Eq. 20. Finally the iterative change in Cauchy stress tensor is obtained by updating the incremental change of effective plastic strain Eq. 19 and substituting it into Eq. 18.

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The implemented non-AFR material model is evaluated by comparing experimental tensile data for DC06 at different material orientations and simulations based on single element uniaxial tension loading. Using an AFR model based on Hill'48 (but using f_p instead of f_y in Eq.3), indeed results in a good agreement in terms of directional strain ratios (Lankford) as seen in Fig. 3a. However poor results are obtained for directional yield stresses. On the other hand, using the stress based yield function (Eq.3) can ensure accurate prediction of directional yield stresses. In case of the DC06, directional yield stresses (Fig. 2) may represent an almost isotropic behavior. However the strain ratios reveal indispensable anisotropic behaviour. Therefore using the yield stress to describe the anisotropic plastic behaviour of material may result in a severe discrepancy between experimental and simulation results. Similar discrepancy is observed for A12090-T3 in work of Cvitanic et al [11]. This issue can be solved by using plastic potential function and yield stress to control the flow and yield of material, respectively. Both directional Lankford coefficients and yield stresses are in good agreement with experimental results. Therefore using non-AFR for case of classical Hill's anisotropic behavior improves the simulation results.

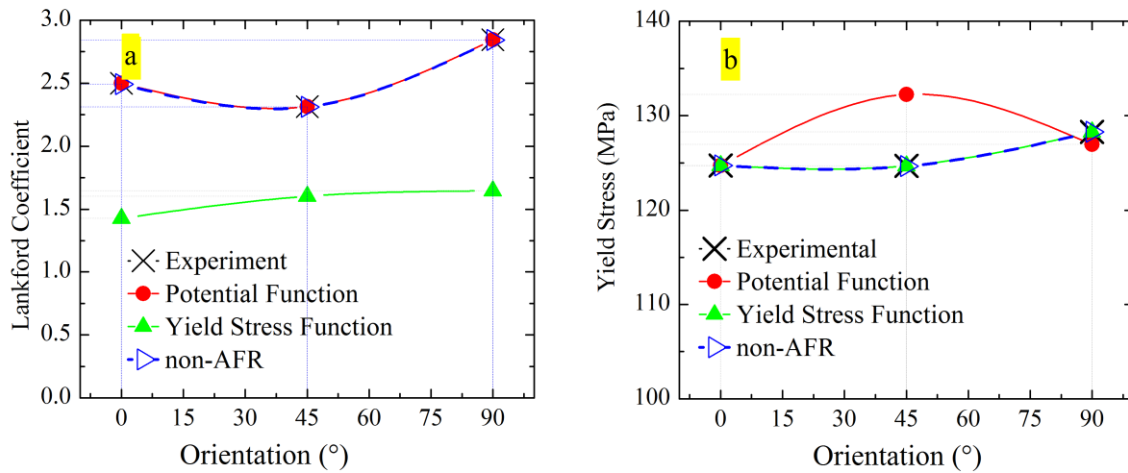


Figure 3. Comparison of simulated and experimental results at different orientations

a) Lankford coefficients, b) yield stresses

ACKNOWLEDGMENT

The financial support from the Ghent University Research Fund (BOF08/24J/106) is greatly appreciated.

References

- [1] Stoughton, T.B., A non-associated flow rule for sheet metal forming. *International Journal of Plasticity*, 2002. **18**(5-6): p. 687-714.
- [2] Safaei, M., W. De Waele, and S.L. Zang, *A rate-independent non-associated constitutive model for finite element simulation of sheet metal forming*, in *International Symposium On Plasticity (Accepted)*, A.S. Khan, Editor 2012: San Juan, PR, USA.
- [3] Spitzig, W.A. and O. Richmond, *The effect of pressure on the flow-stress of metals*. *Acta metallurgica*, 1984. **32**(3): p. 457-463.
- [4] Stoughton, T.B. and J.W. Yoon, *A pressure-sensitive yield criterion under a non-associated flow rule for sheet metal forming*. *International Journal of Plasticity*, 2004. **20**(4-5): p. 705-731.
- [5] Kocks, U.F., C.N. Tome, and H.-R. Wenk, *Texture and anisotropy : preferred orientations in polycrystals and their effect on materials properties*. Cambridge : Cambridge University Press, 1998 (2000 [printing]). xii, 676 p.
- [6] Cardoso, R.P.R. and J.W. Yoon, *Stress integration method for a nonlinear kinematic/isotropic hardening model and its characterization based on polycrystal plasticity*. *International Journal of Plasticity*, 2009. **25**(9): p. 1684-1710.
- [7] Butuc, M.C., et al., *Analysis of sheet metal formability through isotropic and kinematic hardening models*. *European Journal of Mechanics a-Solids*, 2011. **30**(4): p. 532-546.
- [8] Flores, P., et al., *Model identification and FE simulations: Effect of different yield loci and hardening laws in sheet forming*. *International Journal of Plasticity*, 2007. **23**(3): p. 420-449.
- [9] Stoughton, T.B. and J.W. Yoon, *Anisotropic hardening and non-associated flow in proportional loading of sheet metals*. *International Journal of Plasticity*, 2009. **25**(9): p. 1777-1817.
- [10] Taherizadeh, A., D.E. Green, and J.W. Yoon, *Evaluation of advanced anisotropic models with mixed hardening for general associated and non-associated flow metal plasticity*. *International Journal of Plasticity*, 2011. **27**(11): p. 1781-1802.
- [11] Cvitanic, V., F. Vlak, and Z. Lozina, *A finite element formulation based on non-associated plasticity for sheet metal forming*. *International Journal of Plasticity*, 2008. **24**(4): p. 646-687.
- [12] Yoon, J.W., T.B. Stoughton, and R.E. Dick, *Earing prediction in cup drawing based on non-associated flow rule*, in *NUMIFORM '07: Materials Processing and Design: Modeling, Simulation and Applications, Pts I and II*, J.M.A. CeasarDeSa and A.D. Santos, Editors. 2007, Amer Inst Physics: Melville. p. 685-690.