DESIGN OF A NONLINEAR VIBRATION ABSORBER

Maxime Geeroms, Laurens Marijns, Mia Loccufier and Dirk Aeyels

Ghent University, Department EESA, Belgium

Abstract Linear vibration absorbers can only capture certain discrete frequencies. Therefore the use of nonlinear vibration absorbers which can capture a whole range of frequencies is investigated as an alternative. Such a nonlinear vibration absorber has some special characteristics. For example there is a certain frequency-energy dependence. To investigate nonlinear dynamical systems there is a need for new methods. The harmonic balance method is such a method and is discussed. The idea is to substitute a Fourier series expansion of the solution variables into the system equations and 'balance' them. Furthermore two realisations of a nonlinear energy sink as an example of a nonlinear vibration absorber are discussed. One based on the restoring force in a wire, the other one by forcing a linear spring to follow a certain path. As will be discussed, an analog principle can be used for the realisation of a Duffing type of nonlinear absorber.

Keywords: nonlinear; vibration absorber; energy sink; Duffing; harmonic balance;

1 INTRODUCTION

Vibrations in mechanical systems are a source of noise, damage, imprecise movement ... Firstly, one seeks to solutions in the sense of removing the source of vibrations or altering the structure. Often, this is not possible, therefore one wishes to reduce the vibrations.

The most classic solution is the use of a single degree of freedom lightweight linear mass-spring-damper system attached to the structure. This system is also known as the linear vibration absorber for which many methods for tuning have been described in the literature. An important downside of this device is that it is tuned to a single frequency and therefore is not able to reduce multiple vibration modes. This also means that it loses efficiency when the mode of vibration is not known very well.

An alternative is to attach systems that allow multimodal vibration reduction. Examples are the use of multiple linear vibration absorbers each tuned to a different frequency and multi-degree-of-freedom linear vibration absorbers. Both solutions are not always suitable because sometimes the available space to attach auxiliary systems is limited. Therefore the interest in strongly nonlinear vibration absorbers has grown.

Two examples of a nonlinear vibration absorber are the nonlinear energy sink (NES) and the Duffing type of nonlinear absorber. The NES has a pure cubic force-displacement relation where the Duffing type is a combination of a pure cubic and a linear force-displacement relation. Both are capable of resonating at any frequency, making a multimodal vibration reduction possible.

This paper is a half-term report of a thesis in which both type of absorbers and their abilities to reduce vibrations, are investigated through simulation and in practice with a real-life test setup.

2 HARMONIC BALANCE METHOD

One way to investigate the nonlinear dynamical systems under periodic forcing where the response is periodic in time, is the harmonic balance method. There are many variations of this method in literature [3][5]. Here the classical approach is considered. One substitutes a Fourier series expansion of the solution variables into the system equations. Next, one can 'balance' the equations. It means that the terms associated with each harmonic are stated equal to each other. If N_h harmonics are used in our Fourier series expansion, it means that there are $2N_h + 1$ equations for the $2N_h + 1$ harmonic coefficients. When only the fundamental harmonics are used ($N_h = 1$), this method is better known as the HB1 method. One can apply this method to the Duffing type absorber.

$$m\ddot{x} + c\dot{x} + k_{lin}x + k_{nonlin}x^3 = Asin(\omega t) \qquad \text{with} \qquad x(t) = a_0 + \Sigma_{n=1}^{N}\left(a_n cos(n\omega t) + b_n sin(n\omega t)\right) \tag{1}$$

Equating the coefficients associated with each harmonic components $cos(n\omega t)$ and $sin(n\omega t)$ results in solving the set of nonlinear equations (2).

$$\begin{aligned} -m\omega^2 a_1 + c\omega b_1 + k_{lin}a_1 + \frac{1}{2}k_{nonlin}\left[3a_1^3/2 + 3a_1b_1^2/2\right] &= A \\ -m\omega^2 b_1 - c\omega a_1 + k_{lin}b_1 + \frac{1}{2}k_{nonlin}\left[3b_1a_1^2/2 + 3b_1^3/2\right] &= 0 \end{aligned} \tag{2}$$

The amplitude of the response of a nonlinear oscillator changes suddenly at a critical excitation frequency. An own matlab program with a newton-raphson algorithm to solve the set of nonlinear equations (2) managed to get the amplitude versus frequency plot of figure 1. The amplitude is defined as $\sqrt{{a_1}^2 + {b_1}^2}$ and ω_0 is defined as $\sqrt{\frac{k_{lin}}{m}}$.

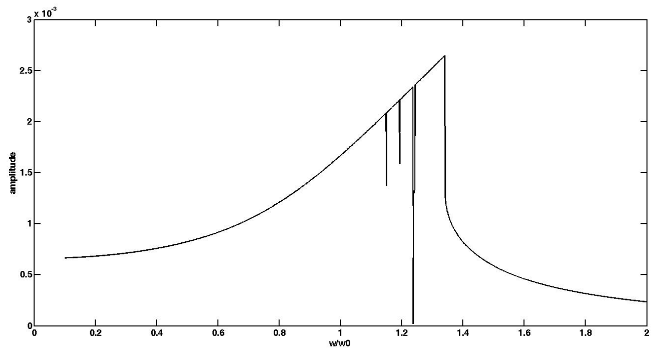

Figure 1. Classical harmonic balance method with $N_h = 1$

Normally, one has to be able to see phenomena like hysteresis in the amplitude versus frequency plot. This can not be seen in figure 1. It failed to show that in the matlab simulation. If many frequency components are taken into account, it is highly possible for HBM to fail. Also, for more complex systems than the duffing oscillator, the HBM may be hard to implement. A downside of the harmonic balance method is that there are computational limits for some strongly nonlinear systems. It can fail to provide accurate predictions for some harmonic components. There are other methods with less computational restrictions, the nonlinear output frequency response function for example. This method can give more accurate harmonic components. However with this method, one can not capture the well known jump phenomenon.

3 FREQUENCY ENERGY DEPENDENCE NES

For the undamped linear absorber, one has the tuning principle of equation (3).

$$\omega_a = \sqrt{\frac{k_{lin}}{m_a}} \approx \sqrt{\frac{k}{m}} \tag{3}$$

Where k and k_{lin} are respectively the spring constants of structure and absorber and m and m_a are respectively the masses of the structure and absorber. For the NES, we can define an equivalent natural frequency and an equivalent tuning principle by using the HB1 method on the undamped equation of motion.

$$m_{nes}\ddot{x}_{nes} + k_{nonlin}x_{nes}^3 = 0 \tag{4}$$

Substituting $x_{nes}(t) = \frac{A}{\omega} sin\omega t$ in equation (4) and balancing the fundamental harmonics($\dot{x}_{nes}(0) = A, x_{nes}(0) = 0$) one obtains an equivalent natural frequency [1].

$$\omega_{eq} = \left[\frac{3}{4} \frac{k_{nonlin} A^2}{m_{nes}} \right]^{\frac{1}{4}} \tag{5}$$

One can clearly see that the NES exhibits a frequency-energy dependence. One can see that ω_{eq} will increase with energy for the NES. This is known as the hardening spring characteristic.

Notice that theory above can only explain certain phenomena. More phenomena inherent at nonlinear systems are present. At a certain level of energy input, a bifurcation occurs and the absorber becomes effective. For multimodal vibrations, each vibration mode features a different energy threshold. Below this level of energy input, the vibration absorption of the NES is very poor and the Duffing type absorber behaves like a linear absorber. This can be demonstrated through simulation. Suppose an SDOF system, attached to the ground with a linear spring. The vibrations of the system will be reduced by a NES. The system parameters are given in table 1. The main system has a natural frequency($\omega = \sqrt{\frac{K}{M}}$) of 19,1 rad/s.

Table 1. System parameters

M	2,5 kg
m	0,5 kg
K	909,6 N/m
$k_{nonlinear}$	1800 000 N/m^3

With the set of parameters in table 1, there is an energy treshold at the initial condition $\dot{x}(0) = 0,08$ (x being the displacement of the main structure). Simulations demonstrate this treshold. In figure 2.a and 2.b, one can clearly see the difference in absorber movement.

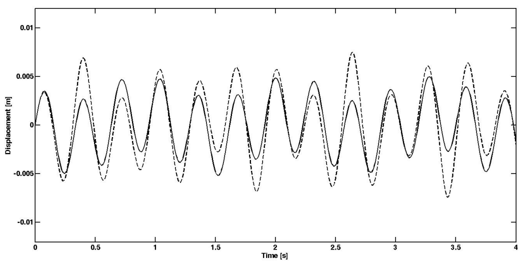

Figure 2.a. $\dot{x}(0) = 0,07$

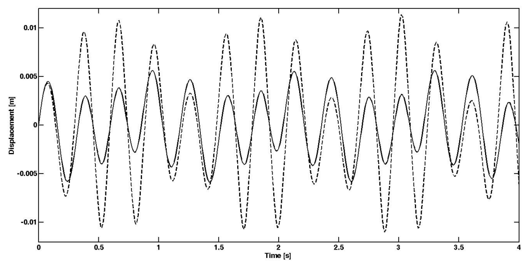

Figure 2.b. $\dot{x}(0) = 0,09$

Figure 2. Main system and absorber response

4 REALISATION OF A CUBIC FORCE-DISPLACEMENT RELATION

Two possible ways of realising a cubic force-displacement relation have been studied. Both realisations are discussed in this section.

4.1 Realisation 1

For the first realisation, a wire, clamped at both sides, is transversly deformed in its center [2]. One obtains a geometry as shown in figure 3.

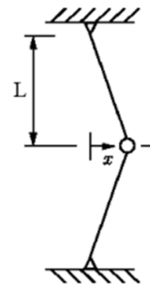

Figure 3. Geometry wire

One can prove that the force F as function of the transverse displacement x can be written as in equation 6.

$$F(x) = 2\left(T_0 + EA\frac{\sqrt{L^2+x^2}-L}{L}\right)\frac{x}{\sqrt{L^2+x^2}} \tag{6}$$

In this equation, the parameter T_0 represents the pretension in the wire. Parameters E, A and L are representing respectively the elasticity modulus, the cross section and the length of the wire. After Taylor expansion one can neglect higer order terms, for sufficient high values of L compared to x, and obtain equation 7.

$$F(x) = \frac{2T_0}{L}x - \frac{T_0 - EA}{L^3}x^3 \tag{7}$$

One can clearly identify a linear term and a cubic term in the force-displacement relation. So a Duffing type of absorber can be realised by attaching a mass to the center of the wire. When the pretension T_0 is zero one can obtain a NES. The nonlinear spring constant is given by $\frac{T_0-EA}{L^3}$ and can be influenced by three independent parameters.

4.2 Realisation 2

For the second realisation a linear spring is forced to follow a certain path as shown in figure 4.

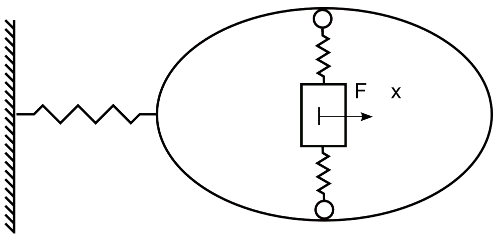

Figure 4. Forced path of a linear spring

One can investigate the force exerted on the main system by the absorber and prove that the force F as function of the transverse displacement x can be written as in equation (8).

$$F = n_p\left(k_{lin}f(x) + T_0\right)\frac{df(x)}{dx} \tag{8}$$

In equation (8), the parameter n_p is a factor which can be one or two when respectively one end of the linear spring or both ends of the linear spring follows a path that differs from a straight line. The parameters k_{lin} and T_0 represents respectively the spring constant of and the pretension in the linear spring. At last $f(x)$ represents the path followed by one or both of the ends of the spring. If we choose $f(x) = ax^2$ with a strictly positive, one obtains equation (9).

$$F = 2aT_0x + 2n_pk_{lin}ax^3 \tag{9}$$

As in the previous section one can clearly identify a linear term and a cubic term in the force-displacement relation. By setting the pretension T_0 to zero, one can obtain a pure cubic force-displacement relation. The nonlinear spring consant is given by $2n_pk_{lin}a$ and can be influenced by three independent parameters. As in the previous section a Duffing type of absorber or a NES can be realised.

5 CONCLUSION

This paper discussed the harmonic balance method as a tool to predict the output amplitude when the exitation of a nonlinear dynamic system is periodic in time. Typical phenomena for nonlinear systems like the jump phenomenon, can be visualised. A major downside of the harmonic balancing method is that the set of nonlinear equations will become complex when more harmonics are used. Two ways of realising a cubic force-displacement relation were described. Both realisations will be implemented in practice. They will be used to obtain insight in nonlinear vibration reduction and to link the theory to practice.

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7 REFERENCES

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