Power and satisfaction analysis :

An application to the Belgian House of Representatives

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I. Introduction.

This paper deals with numerical indices which measure the position of a player or a voter in a game. We distinguish power and satisfaction indices. The former class measures the ability of a player to change the outcome of a game by changing his vote. The latter measures to which extent players agree with the outcome of a game, regardless their ability to control the game. Finally, an index has been constructed which gives the probability that a player will join a minimal winning coalition. In the context of a political game, this concept can be interpreted as the probability that a political party will join a government coalition.

The framework above has been applied on post-war election results for the Belgian parliament (1).

II. Weighted voting games.

Let us introduce some elementary notions of the theory of weighted voting games. The game is played by a set of players, called parties. The set is labelled N with parties numbered 1,2,...,n. A coalition D is a subset of N = [1,2,...,n]. A weighted voting game G (2) is defined by a (n+1)-tuple

$$[q\ ;\ v_1,\ v_2,\ldots,v_n]$$

where $(v_1+v_2+\ldots+v_n)/2 < q \leqslant v_1+v_2+\ldots+v_n$.

The non-negative integers v_i are the weights of the n players, which we can take to be the number of seats in the house of representatives.

(1) L. LAUWERS, P. UYTTERHOEVEN, *Belgische politieke partijen in de na-oorlogse periode : coalitiekracht en Shapley-waarde*. Leuven, 1986.

(2) P.D. STRAFFIN, Probability Models for Power Indices, in : P. ORDESHOOK (Ed.), *Game Theory and Political Science*. New York, 1978, pp. 477-510.

The positive integer q is called the quota. This quota be interpreted as the threshold to make a coalition winning. The set W of winning coalitions is defined by

$$W = [D \subseteq N \mid \sum_{j \in D} v_j \geqslant q].$$

Thus, if the set of players who vote « yes » on a certain alternative has a weight greater than or equal to the quota q the alternative passes. If for a player i, $v_i \geqslant q$ then player i is said to be a dictator.
Let $L = DN\text{-}W = [D \subseteq N \mid \sum_j v_j < q]$ be the set of losing coalitions. A coalition D whose complement N-D is losing is called a blocking coalition. The set of blocking coalitions will be denoted by B. In weighted voting games $W \subseteq B$. Coalitions D for which neither D nor N-D are winning are said to be strictly blocking. They will be gathered in the set SB. Denote by W_i the set of winning coalitions containing player i. The sets L_i, B_i, SB_i are defined analogously.

A winning coalition D containing player i such that D-[i] is losing is said to be a swing for player i. The set of swings for player i is denoted by S_i. A player without swing is a dummy player.

A winning coalition D is defined to be a minimal winning coalition if D is a swing for all players $i \varepsilon D$:

Thus a set of players is a minimal winning coalition if every player in the set is needed to make the set winning.

A player who appears in every minimal winning coalition is called a veto player. Note that a dictator is also a veto player.

Small letters corresponding to the above defined sets will stand for their cardinality :

n = number of players
w = number of winning coalition $= \# W$
$w_i = \# [D \varepsilon W / i \varepsilon D]$
$l = \# [D \subseteq N / D \text{-}\varepsilon W] = \# L$
$l_i = \# [D \varepsilon L / i \varepsilon D]$
$b = \# [D \subseteq N / N\text{-}D \text{-}\varepsilon W] = \# B$
$b_i = \# [D \varepsilon B / i \varepsilon D]$
$sb = \# [D \subseteq N / D \text{-}\varepsilon W \text{ and } N\text{-}D \text{-}\varepsilon W]$
$sb_i = \# [D \varepsilon SB / i \varepsilon SB]$
$s_i = \# [D \varepsilon W_i / D\text{-}[i] \text{-}\varepsilon W]$
$mw = \# [D \varepsilon W / \bar{V} i : i \varepsilon D \rightarrow D\text{-}[i] \text{-}\varepsilon W]$
$mw_i = \# [D \varepsilon MW / i \varepsilon D]$,

where $\text{-}\varepsilon$ stands for the negation of ε and means « is no element of » and where $\bar{V}$ is used for the universal quantifier « for all ».

Note that $l + w = 2^n$, $l_i + w_i = 2^{n-1}$ and that $l = b$.

As an example we will applicate the theory to the Belgian House of Representatives. Consider the situation in 1949 :

(1) CDEM	(2) SOC	(3) LIB	(4) COMM
105	66	29	12

(cf table I)

with quota $q = (105 + 66 + 29 + 12)/2 + 1 = 107$.

This game will be denoted by :

$$(107 ; 105, 66, 29, 12)$$

For $v_1 + v_3 + v_4 = 105 + 29 + 12 \geqslant 107$ the set [1,3,4] is a winning coalition. Since the set [3,4] is a loosing coalition, we can conclude that [1,3,4] is a swing for player 1 (i.e. CDEM), thus in the set [1,3,4] player 1 is needed to make the coalition a winning one.

Verify also that :

— [1,2], [1,2,3], [1,2,4], [1,2,3,4], [2,3,4] are all the sets in W_2 ;
— [1,2] and [2,3,4] are the two sets of S_2 ;
— MW = [[1,2], [1,3], [1,4], [2,3,4]] such that $mw = 4$, $mw_1 = 3$, $mw_2 = mw_3 = mw_4 = 2$.

III. Indices of power.

A power index measures the ability of a player to force an alternative by voting for it. Because dummy players are redundant in every winning coalition, this means that power indices vanish for dummy players. At the other extreme they attribute unit power for dictators. Since a player i is able to be decisive in some coalition D if and only if D is a swing for i, power indices are normalizations of s_i.

The absolute Banzhof index (1965-1968) (3)

$$AB(i) = s_i/2^{n-1}$$

measures the likelihood that a coalition is a swing for player i to a coalition containing i. To compute this index run through the subsets D of N containing i, and pick out those subsets which are winning and where player i is needed to make it a winning coalition. The number of such sets is s_i. AB(i) can be interpreted as the probability that player i casts

(3) R. DUBEY, L. SHAPLEY, Mathematical properties of the Banzhof power index, in : *Rand-paper P-6016*, The Rand Corporation, 1977.

a critical vote (i.e. a vote that changes a losing coalition into a winning one) assuming that all coalitions of the n—1 remaining players are equally likely. The assumption is equivalent to each player having probability 1/2 of voting for a given alternative. This assumption will be made all through this text.

The relative Banzhof index

$$RB(i) = s_i \,/ \sum_{j=1}^{n} s_j$$

is a player's proportion of swings.

Coleman (1971) introduces two absolute indices of power :

$$CP(i) = s_i/w$$
$$CI(i) = s_i/(2^n\text{-}w) = s_i/b.$$

CP(i) is interpreted as the probability that an arbitrary winning coalition is a swing for player i, or as the proportion of times that a player can block the action of a winning coalition by withdrawing from it. It measures the power of a player to prevent action. Note that CP = 1 for veto players.

In the same way CI measures the power of a player to initiate action. CI is the proportion of times that a player changes a non-winning coalition into a winning one by joining it.

LEMMA III.1 : In a weighted voting game $CP(i) \geqslant AB(i) \geqslant CI(i)$. If there are no strictly blocking coalitions then $CP(i) = AB(i) = CI(i)$.

PROOF. The power set DN is the disjoint union of W and L. If there is no strickly blocking then $W \xrightarrow{\text{COMPL}} L : D \mapsto N\text{-}D$ is a bijection between W and L. So $2^n = 2w$ and $w = 2^{n-1}$.
Q.E.D.

At last consider :

$$P(i) = mw_i/mw.$$

This index can be interpreted as the number of times that a minimal winning coalition contains a player i. P is a measure for the probability that a player participates a minimal winning coalition. It is clear that P(i) = 1 if and only if i is a veto player.

In our example $AB(2) = s_2/2^3 = 0.25$ and since there is no strictly blocking $AB(2) = CP(2) = CI(2)$. For $mw = 4$ and $mw_2 = 2$ we have $P(2) = 0.5$.

IV. **Indices of satisfaction.**

As power indices provide different measures of the ability of a player to change the outcome by changing his vote, a satisfaction index will measure the probability that a player agrees with the outcome.

Two indices are currently used :

the Zipke index (4)

$$Z(i) = w_i/2^{n-1}$$

the Brams-Lake index

$$BL(i) = (w_i + b_i)/2^n$$

To interprete the first one assumes that a player gets unit satisfaction if he is in a winning coalition and zero otherwise.

The Brams-Lake index is based on another concept of satisfaction : a player gets unit satisfaction if he votes with an alternative and it wins or if he votes against it and it loses.

The indices can be regarded as the probability a player is satisfied under the corresponding concepts. For dummy players BL $= 1/2$ and Z $= 1/2$. Because in weighted voting games $b_i = w_i + sb_i$, we have

LEMMA IV.1 : In weighted voting games $Z(i) \leqslant BL(i)$ and if there is no strictly blocking then $Z(i) = BL(i)$.

In the example $Z(2) = BL(2) = w_2/2^3 = 5/8$.

For the sake of completeness we will mention a theorem of Brams-Lake (5) on the connection between power- and satisfaction-indices :

THEOREM IV.2 : $AB(i) = 2(BL(i) - 1/2) = 2(Z(i) - w/2^n)$.

This theorem shows that the notion of swing is not needed to define power indices.

This short introduction into weighted voting games will be concluded with two remarkable examples :

G_1	RB	Z	BL	G_2	RB	Z	BL
3	1	1	1	3	1/3	1/4	5/8
1	0	1/2	1/2	1	1/3	1/4	5/8
1	0	1/2	1/2	1	1/3	1/4	5/8

(4) C.H. NEVISON *et al.*, A Naive Approach to the Banzhof Index of Power, in : *Behavioral Science*, 1978 (XXIII), pp. 130-131.

(5) S.J. BRAMS, M. LAKE, Power and Satisfaction In a Representative Democracy, in : P. ORDESHOOK (Ed.), *Game Theory and Political Science*. New York, 1978, pp. 529-562.

Compare these two games and note that although the dummy players gain in power and in Brams-Lake satisfaction, their Zipke-satisfaction decrease. This is due to the fact that the concept of Zipke-satisfaction considers only winning and no blocking situations, and of course if W is small Z will be small.

Another kind of paradox does appear in the following situation

$$G_1 = [12;5,5,5,3,3] \quad G_2 = [12;7,5,5,2,2]$$

G_1	RB	AB	P	G_2	RB	AB	P
5	7/27	7/16	.714	7	9/25	9/16	.5
5	7/27	7/16	.714	5	7/25	7/16	.750
5	7/27	7/16	.714	5	7/25	7/16	.750
3	3/27	3/16	.428	2	1/25	1/16	.250
3	3/27	3/16	.428	2	1/25	1/16	.250

The power indices for player 1 increase while his participation index decreases.

V. **An application to the Belgian House of Representatives.**

Using post-war election results for the Belgian parliament, we can compute power and satisfaction of political parties. The following abbreviations will be used :

CDEM	:	Christian-democrats
SOC	:	Socialists
LIB	:	Liberals
VU	:	Volksunie
FDF-RW	:	Fédération des francophones et Rassemblement Wallon
COMM	:	Communists
ECOL	:	Ecologists
VLBL	:	Vlaams Blok
RAD-UDRT	:	Union Démocratique pour le Respect du Travail.

Two weighted voting games are considered :

- If a government wants to change the constitution, a 2/3-majority is required. In that case q=142 (the total number of seats is 212).
- In the other cases the required voting quota is q=107.

The evolution of the composition of the parliament is given in table I.

TABLE I

The composition of the House of Representatives

	CDEM	SOC	LIB	VU	FDF-RW	COMM	ECOL	VLBL	RAD-UDRT
1949	105	66	29			12			
1950	108	77	20			7			
1954	95	86	25			4			
1958	104	84	21	1		2			
1961	96	84	20	5		5			
1965	77	64	48	12	5	6			
1968	69	59	47	20	12	5			
1971	67	61	34	21	24	5			
1974	72	59	30	22	25	4			
1977	80	62	33	20	15	2			
1978	82	58	37	14	15	4		1	1
1981	61	61	52	20	8	2	4	1	3
1985	69	67	46	16	3	0	9	1	1

Calculated absolute power values are presented in table II. If no strictly blocking coalitions occur, absolute Banzhof power and Coleman power are equal. Otherwise, CP(i) > AB(i) > CI(i), as proved in section III. The normalized Banzhof index is given in table III.

TABLE II

The absolute Banzhof power AB (1), the Coleman index CP (2) and the Coleman index CI (3) (q = 107)

		CDEM	SOC	LIB	VU	FDF-RW	COMM	ECOL	VLBL	RAD-UDRT
1949		.750	.250	.250			.250			
1950		1	0	0			0			
1954		.500	.500	.500	0		0			
1958	1	.687	.312	.312	.062		.187			
	2	.733	.333	.333	.066		.200			
	3	.647	.294	.294	.059		.177			
1961	1	.609	.391	.391	.109		.109			
	2	.619	.397	.397	.111		.111			
	3	.600	.385	.385	.108		.108			
1965		.500	.500	.500	0	0	0			
1968	1	.531	.469	.469	.031	.031	.031			
	2	.548	.484	.484	.032	.032	.032			
	3	.515	.455	.455	.030	.030	.030			
1971	1	.531	.469	.281	.219	.219	.031			
	2	.548	.484	.290	.226	.226	.032			
	3	.515	.455	.273	.212	.212	.030			
1974	1	.531	.469	.281	.219	.219	.031			
	2	.544	.484	.290	.226	.226	.031			
	3	.485	.485	.242	.242	.242	.032			
1977		.625	.375	.375	.125	.125	0			
1978		.625	.375	.375	.125	.125	0		0	0
1981		.500	.500	.500	0	0	0	0	0	0
1985		.500	.500	.500	0	0	0	0	0	0

TABLE III

Relative Banzhof power (q = 107)

	CDEM	*SOC*	*LIB*	*VU*	*FDF-RW*	*COMM*	*ECOL*	*VLBL*	*RAD-UDRT*
1949	.500	.167	.167			.167			
1950	1	0	0			0			
1954	.333	.333	.333			0			
1958	.440	.200	.200	.040		.120			
1961	.371	.238	.238	.067		.067			
1965	.333	.333	.333	0	0	0			
1968	.340	.300	.300	.020	.020	.020			
1971	.304	.268	.161	.125	.125	.018			
1974	.304	.268	.161	.125	.125	.018			
1977	.385	.231	.231	.077	.077	0			
1978	.385	.231	.231	.077	.077	0		0	0
1981	.333	.333	.333	0	0	0	0	0	0
1985	.333	.333	.333	0	0	0	0	0	0

TABLE IV

The absolute Zipke satisfaction index Z(i) [1],
the Brams-Lake satisfaction index [2] and the Zipke index ZP [3] for q = 107

		CDEM	*SOC*	*LIB*	*VU*	*FDF-RW*	*COMM*	*ECOL*	*VLBL*	*RAD-UDRT*
1949		.875	.625	.625			.625			
1950		1	.500	.500			.500			
1954		.750	.750	.750	.500		.500			
1958	1	.812	.625	.625	.500		.562			
	2	.844	.656	.656	.531		.594			
	3	.867	.667	.667	.533		.600			
1961	1	.797	.687	.687	.547		.547			
	2	.805	.695	.695	.555		.555			
	3	.809	.698	.698	.555		.555			
1965		.750	.750	.750	.500	.500	.500			
1968	1	.750	.719	.719	.500	.500	.500			
	2	.766	.734	.640	.609	.609	.516			
	3	.774	.742	.742	.516	.516	.516			
1971	1	.750	.719	.625	.594	.594	.500			
	2	.766	.734	.734	.516	.516	.516			
	3	.774	.742	.742	.516	.516	.516			
1974	1	.750	.719	.625	.594	.594	.500			
	2	.766	.734	.640	.609	.609	.516			
	3	.774	.742	.645	.613	.613	.516			
1977		.812	.687	.687	.562	.562	.500			
1978		.812	.687	.687	.562	.562	.500		.500	.500
1981		.750	.750	.750	.500	.500	.500	.500	.500	.500
1985		.750	.750	.750	.500	.500	.500	.500	.500	.500

Since the 1981 elections only CDEM, SOC and LIB represent coalition power. No other party has any influence. Despite notable election gains in 1981 (6 seats), the coalition power of the VU was reduced to zero. In 1985 the christian-democrats and socialists improved remarkably (respectively 8 and 6 seats), but power relations didn't change. One can note that with exception of 1971 and 1974 the power of liberals and socialist was equal, despite the fact that the socialists possess much more seats in the House of Representatives.

Table IV compares the absolute Zipke satisfaction index $Z(i) = w_i/2^{n-1}$ and the Brams-Lake index.

In accordance with the Coleman power measure CP(i), a similar Zipke index $ZP = w_i/w$ is constructed. It has to be interpreted as the probability that an arbitrary winning coalition is containing i. Clearly, if strictly blocking occurs, then $ZP(i) > z(i)$.

As a consequence of theorem IV.2, a dummy player has minimal satisfaction $z(i) = ZP(i) = BL(i) = 1/2$. Political parties with no Banzhof (or Coleman) power have also minimal satisfaction.

Special emphasis is on the P(i)-index defined above. Under the assumption that a government will be a minimal winning coalition, one can interprete this index as the probability that a party will join government. Table V contains a complete list of all government coalitions since 1949,

TABLE V

Government coalitions since 1949

1. G. Eyskens	11.08.1949-06.06.1950	CDEM, LIB	MWC
2. J. Duvieusart	08.06.1950-11.08.1950	CDEM	MWC
3. J. Pholien	16 08.1950-09.01.1952	CDEM	MWC
4. J. Van Houtte . . .	15.01 1952-12.04 1954	CDEM	MWC
5. A. Van Acker . . .	22.04.1954-02 06 1958	SOC, LIB	MIN
6. G. Eyskens	23.06.1958-27.03.1958	CDEM	MIN
7. G. Eyskens	06.11.1958-27.03.1961	CDEM, LIB	MWC
8. T. Lefèvre	25.04.1961-24 05 1965	CDEM, SOC	MWC
9. P. Harmel	27.07 1965-11.02 1066	CDEM, SOC	MWC
10. P. Van den Boeynants	19 03.1966-07.02.1968	CDEM, LIB	MWC
11. G. Eyskens	17.06.1968-08.11.1971	CDEM, SOC	MWC
12. G. Eyskens	30.01.1972-22.11.1972	CDEM, SOC	MWC
13. E. Leburton	26 01.1973-19.01.1974	SOC, CDEM, LIB	MAJ (MWC)
14. L. Tindemans . . .	25.04 1974-11.06.1974	CDEM, LIB	MIN
15. L. Tindemans . . .	11.06.1974-04 03.1977	CDEM, LIB, RW	MWC
16. L. Tindemans . . .	04.06.1977-18.04 1977	CDEM, LIB	MIN
17. L. Tindemans . . .	26.05.1977-11.10 1978	CDEM, SOC, VU, FDF	MAJ
18. P. van den Boeynants	20.10.1978-18 12.1978	CDEM, SOC, VU, FDF	MAJ
19. W. Martens	03.04 1979-23.01.1980	CDEM, SOC, FDF	MAJ (MWC)
20. W. Martens	23 01.1980-09.04 1980	CDEM, SOC	MWC
21. W. Martens . . .	18.05.1980-07.10.1980	CDEM, SOC, LIB	MAJ (MWC)
22. W. Martens	22.10.1980-02.04.1981	CDEM, SOC	MWC
23. M. Eyskens	06 04.1981-21.09.1981	CDEM, SOC	MWC
24. W. Martens	17.12 1981-15.10.1981	CDEM, LIB	MWC
25. W. Martens	28.11.1985	CDEM, LIB	MWC

if all government coalitions are equally likely. If P(i) = 1, then i is a veto-player and he will join every government. We distinguish the following types of government :

MWC	:	a minimal winning coalition w.r.t. q = 107.
MIN	:	a government with minority w.r.t. q = 107.
MAJ	:	a government with 2/3-majority which is not minimal winning w.r.t. q = 142.
MAJ(MWC)	:	a government with 2/3-majority which is minimal winning w.r.t. q = 142.

Most governments are minimal winning. The participation-index is given in table VI.

TABLE VI

The participation index P(i) (q = 107)

	CDEM	*SOC*	*LIB*	*VU*	*FDF-RW*	*COMM*	*ECOL*	*VLBL*	*RAD-UDRT*
1949	.750	.500	.500			.500			
1950	1	0	0			0			
1954	.667	.667	.667	0		0			
1958	.750	.500	.500	.250		.500			
1961	.667	.500	.500	.500		.500			
1965	.667	.667	.667	0	0	0			
1968	.400	.800	.800	.200	.200	.200			
1971	.571	.571	.571	.571	.571	.143			
1974	.571	.571	.571	.571	.571	.143			
1977	.600	.600	.600	.400	.400	0			
1978	.600	.600	.600	.400	.400	0		0	0
1981	.667	.667	.667	0	0	0	0	0	0
1985	.667	.667	.667	0	0	0	0	0	0

It is interesting to compare table III and VI. Despite the fact that Christian-democrats frequently have a higher Banzhof power, several parties have the same probability to join the government. In the period 1971-1977, the participation probability of all parties except the communist party was equal.

One also notes a paradox in 1968. In spite of a larger Banzhof value, the probability of CDEM to join government was nevertheless smaller than that of SOC and LIB. Such a paradox happens if the smaller parties have the opportunity to form minimal winning coalitions which exclude the larger party.

An interesting paradox occurred in the 1981 elections. The Christian-democrats lost 21 seats. Accordingly the Banzhof power decreased, but the participation probability increased. Since 1981 the traditional parties CDEM, SOC and LIB have equal Banzhof power and participation probability.

One can calculate the critical number of seats a party can lose without changing its participation probability P(i). This critical number is equal to

$$C = \min_{D \in B} \big|\, |D| - q \,\big|, \text{ where B is the set of blocking coalitions.}$$

For the 1985 election, C = 6. The government coalition CDEM, LIB therefore can lose 6 seats without affecting its P(i)-value. But even if more than 6 seats are lost, the present government parties preserve the same Banzhof power and participation probability in a wide range of possible shifts of seats. The following hypothetical shifts for example do not alter Banzhof power and participation probability w.r.t. the elections of 1985 :

CDEM	*SOC*	*LIB*	*VU*	*FDF-RW*	*ECOL*	*VLBL*	*RAD-UDRT*
— 7	+ 7						
	+ 7	— 7					
— 7	+ 2		+ 3		+ 2		
— 7	+ 3		+ 3		+ 3	— 1	— 1
	+ 3	— 7	+ 3		+ 3	— 1	— 1
— 8					+ 8		

One has to be very cautious in interpreting results. In the analysis above, one assumes that political parties do not represent ideologies. All parties are ideologically interchangable and each party is considered as ideologically homogeneous. If some fractions within a party can change preferences w.r.t. coalition formation, results will be completely different. In particular the critical number of seats, C=6 in 1985, only makes sense if the likelihood of a CDEM-LIB coalition is unaltered.

Summarizing, one can state that the preceding results has to be tempered w.r.t. ideological distances between and within parties. However, one can deal with ideological differences by bringing in some subjective probability distributions, which reflect beliefs about coalition formation. One can weight possible coalition partners by adjudging subjective probabilities. Another way of attack is to define an associated weighted voting with quarelling (Nevinson [6]). This game is defined by eliminating from the winning sets all coalitions which contain ideologically incompatible parties.

If we define such a game for the 1985 elections, it makes only sense to consider quarelling sets containing the traditional parties because the

(6) C.H. NEVISON *et al.*, Structural Power and Satisfaction in Simple Games, in : S.J. BRAMS *et al.* (Eds), *Applied Game Theory*. Physica-Verlag, 1979, pp. 39-57.

other parties don't have any power, even if they form an alliance. If Socialist and Liberals are incompatible, the participation probability of CDEM increases from .667 to 1, while the P(i)-value of SOC and LIB decreases from .667 to .5.

Table VII and VIII give the relative Banzhof power and participation probability for the voting quota q = 142. Only relevant years are considered.

TABLE VII

Relative Banzhof power for q = 142

	CDEM	*SOC*	*LIB*	*VU*	*FDF-RW*	*COMM*	*ECOL*	*VLBL*	*RAD-UDRT*
1965	.429	.286	.004	.004	.095	.005			
1968	.318	.273	.136	.136	.009	.005			
1971	.341	.295	.114	.114	.114	.002			
1974	.400	.300	.100	.100	.100	0			
1977	.474	.368	.005	.005	.005	0			
1978	.431	.331	.069	.069	.069	.018		.006	.006

TABLE VIII

The participation index P(i) for q = 142

	CDEM	*SOC*	*LIB*	*VU*	*FDF-RW*	*COMM*	*ECOL*	*VLBL*	*RAD-UDRT*
1965	1	.667	.500	.500	.334	.334			
1968	.800	.800	.600	.600	.600	.400			
1971	.800	.800	.600	.600	.600	.200			
1974	1	.750	.500	.500	.500	0			
1977	1	.500	.500	.500	.500	0			
1978	1	.833	.334	.334	.334	.167		.167	.167

W.r.t. q = 142, the liberals have less Banzhof power than the socialists and with exception of 1977 also a lower participation probability. As can be seen from table VIII, the Christian-democrat party often is a veto player w.r.t. q = 142. The participation probability of liberals, VU and FDF-RW is the same with exception of 1968.

VI. **Conclusions.**

Relations among parties can be analysed by power- and satisfaction-indices. Another index, the P(i)-index, provides an adequate measure to judge the probability that a party will join a government. Since the elections of 1981 the traditional parties CDEM, SOC and LIB join the same Banzhof power and participation probability. The other parties have no power and participation value at all.

Ideological differences between parties are ignored and all coalitions have the same likelihood. This assumption clearly reduces the scope of the results, but nevertheless gives insight in the way numerical strenght influence coalitional behaviour of political parties.

Summary : Power and satisfaction analysis : an application to the Belgian House of Representatives.

Using post-war election results for the Belgian House of Representatives, the power relations among political parties are analysed by calculating power- and satisfaction indices. Also, a participation index has been constructed to calculate the probability that a party will join a government coalition.

Since the election of 1981 the traditional parties (christian-democrats, socialists and liberals) join the same Banzhof power and participation probability. The other parties represent no power and participation value at all.

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